Optical pumping method for squeezing and entanglement in the ground-state spin subsystems of macroscopic atomic ensembles

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We describe the mechanism of squeezing and entanglement in the ground-state spin subsystems of macroscopic atomic ensembles via the process of stimulated cooperative Raman-type scattering of correlated photon pairs. This mechanism is analogous to common optical pumping techniques normally used for depopulation of Zeeman sublevels in macroscopic atomic ensembles. We show that, by the excitation of atoms, oriented in their angular momenta, with nonclassical light consisting of a rare flux of strongly correlated photon pairs, essential correlations in the macroscopic spin fluctuations can be transferred and accumulated. In turn this leads to squeezing or entanglement in the quasispin (alignment) subsystems of macroscopic ensembles. We discuss in particular a scheme for an interferometer that would be available for the control or teleportation of quantum states created in quasispin subsystems. Such squeezed and entangled states can be stored in long-lived spin subsystems and can be further used in quantum information protocols based on the continuous variable technique.

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I. INTRODUCTION

Recent experiments on teleportation of quantum states in systems of continuous variables [1,2] motivated certain expectations for their possible applications in the area of quantum information [3–5]. The macroscopic objects physically described as systems of harmonic oscillators seem preferred candidates for any quantum information protocols based on continuous variables. It is quite natural that in the system of harmonic oscillators the transformations between squeezed or entangled states might act as quantum gates, normally associated with the basis of discrete variables. Fluctuations of macroscopic spins or quasispins of an initially spin-oriented atomic ensemble, containing a large number of atoms, can be considered, under certain restrictions, as an example of a harmonic oscillator. In spite of the fact that the commutation rules for collective angular momentum obey the usual rules of Pauli matrices, if we consider small fluctuations scaled with respect to the average spin orientation along the z axis, the fluctuating transverse (x and y) components of the collective spin can be properly described in terms of the quadrature components of a harmonic oscillator. Moreover, in contrast to canonical optical realizations, the so called spin-made harmonic oscillator can be fixed in space and its quantum state can be stored for a very long time due to the negligible role of relaxation processes.

The possibility of spin squeezing was recently demonstrated in the excitation of cesium atoms with a combination of squeezed and classical light [6]. There have been several proposals as to how quantum states in ground-state spin subsystems can be prepared in experiments: via complete absorption of nonclassical light [7], via mapping of the quantum state of light onto a sample with electromagnetically induced transparency [8,9], or via cooperative Raman-type scattering of correlated photon pairs [10]. At least one possible experimental realization of Faraday-type long-distance coupling of ground-state spin subsystems between two spatially separated atomic samples has been reported in [11]. It is important to recognize that the concept of storing quantum information in macroscopic spin fluctuations should not be confused with another quite promising idea: that of storing one qubit of quantum information in a macroscopic ensemble by reduction of its state after single-photon forward Raman scattering as proposed in [12]. An important advantage of macroscopic entangled or squeezed spin states in any realization is that because of the suppression of relaxation processes the entanglement can be created at elevated temperatures [11], thus not requiring specialized samples for its generation and detection.

In the present paper we discuss an optical mechanism, which can be compared with the optical pumping technique, and show how quantum correlations in the ground-state spin subsystems of atomic ensembles can be transferred via Raman-type scattering of weak quantum light. The proposed mechanism is aimed at squeezing or entanglement of the macroscopic spin fluctuations of spin-oriented ensembles. It is based on the important property that the Raman-type scattering of the twin photons of one pair, generated by an optical parametric oscillator far below threshold, becomes preferably cooperative if it is additionally stimulated by a classical mode. This extends our earlier proposal [10] but considered now for more realistic Λ-type excitation in $1\rightarrow 1$ optical transitions, where Raman scattering appears in orthogonal circular polarizations; a good example of such a transition can be found in the hyperfine manifolds of alkali-metal atoms. In this case the squeezing or entanglement should appear between quasispin states, i.e., between alignment components. That is why we discuss in particular a scheme for an interferometer able to control or teleport the quantum states created in quasispin subsystems.

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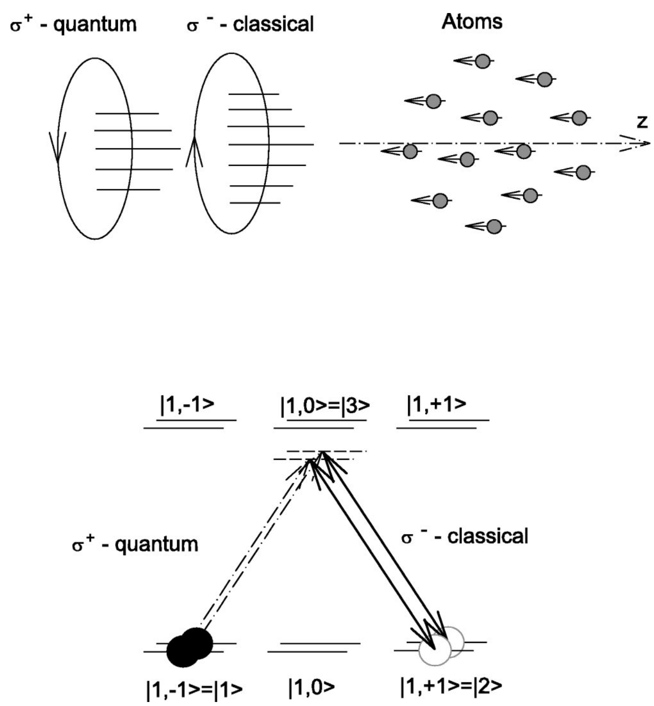


FIG. 1. Geometry of Λ-type excitation of an ensemble of spin oriented atoms by right circular polarized quantum light and by left circular polarized classical light. Correlations existing in the quantum field create double-particle coherence in the ground-state spin subsystem.

II. DIAGRAM CLASSIFICATION OF THE BASIC PROCESSES

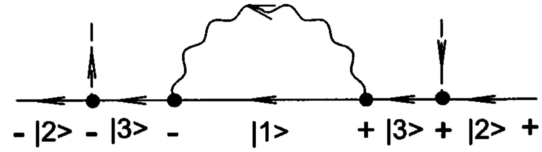
Consider an ensemble of N identical atoms originally 100% polarized in their ground state along a particular quantization direction z and then depumped by Λ-type excitation to other Zeeman sublevels, as shown in Fig. 1. The excitation is initiated by one polarization mode of weak quantum light and the Raman-type transition is additionally stimulated by a strong classical mode in orthogonal polarization. As an example convenient for discussion and available for experimental realization, we will consider below the Λ-type configuration between atomic states characterized by lower $F=1$ and upper $f=1$ angular momenta. This transition can be selected, for example, in the hyperfine manifold of ultracold ^{87}Rb atoms. Then the quantum and classical modes appear in the right and the left circular polarizations, respectively (see Fig. 1). The important feature and advantage of this configuration is that the Zeeman transition $|1,0\rangle \rightarrow |1,0\rangle$ is forbidden and need not be considered further. Then let us denote the Zeeman substates of the Λ configuration as $|1,-1\rangle = |1\rangle$ and $|1,+1\rangle = |2\rangle$ in the lower state and $|1,0\rangle = |3\rangle$ in the upper state.

The original projection operator of the N-particle density matrix of the ensemble can be written as

$$\hat{\rho}_0 = |1,1,\ldots,1\rangle\langle 1,1,\ldots,1|. \tag{2.1}$$

Such a collective density matrix has a separable form and can be factorized into the product of the single-particle density matrices of each atom. If all the atoms of the ensemble are located inside the coherence volume of the quantum mode, then short-term optical excitation modifies their collective density operator to a new form in the process of light scattering. If, as assumed, the classical mode is sufficiently strong, the scattering will be preferably of Raman type and coherent in the direction of propagation of this mode.

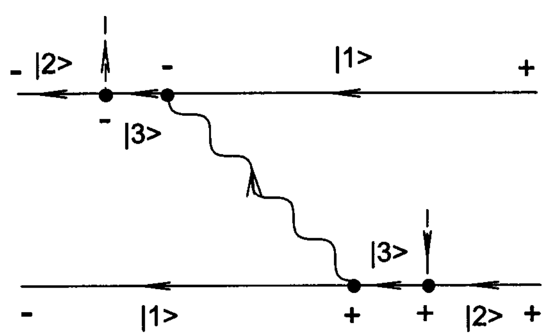
There are several processes contributing to the Raman-type scattering in the initial stage of the evolution. First is the independent scattering on any random atom of the ensemble, which can be described by the following Keldysh-type diagram:



(2.2)

where the internal solid and wavy lines, coupling the vertices of different signs, describe, respectively, the original single-particle density matrix (more precisely, the single-particle Green function) of the ith atom ($i=1$–N) and the single-photon normal-type correlation function of the quantum light. Further evolution of the atomic density matrix up to the observation time is described by retarded (with $-$ sign) or advanced (with $+$ sign) Green functions, and the dashed vertical arrows are the interactions with the classical mode. For other notation regarding the Keldysh technique, we follow the definitions in [13].

At the same order of perturbation theory there are cooperative contributions to the double-particle density matrix of the ensemble, which are described by the diagram



(2.3)

where the solid lines relate to any selected pair of ith and jth atoms with indices i,j running from 1 to N. This kind of radiation coupling extends to all pairs with $i \neq j$, and, as follows from this graph, it leads to different contributions to the double-particle density matrix when the ith atom is associated with the upper (lower) line and the jth atom with the lower (upper) line.

After the interaction time t, following the perturbation theory approach, the above processes lead to the following transformation of the atomic density operator (2.1):

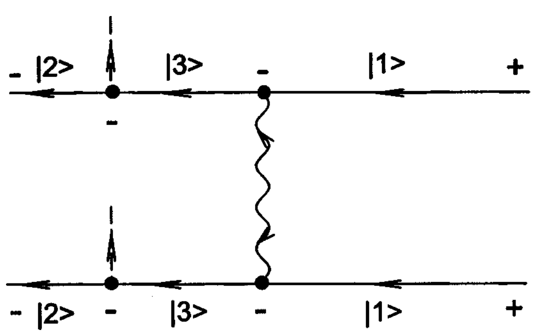
$$\hat{\rho}'(t)=(1-NRt)\hat{\rho}_0+Rt\sum_{i,j=1}^{N}|1,1,\ldots,2_i,\ldots,1\rangle\langle 1,1,\ldots 2_j,\ldots,1|, \tag{2.4}$$

where the rate constant R is responsible for the Raman-type transitions and appears the same for both types of diagrams (2.2) and (2.3). It is given by

$$R\sim\frac{\Omega_R^2|\Omega'|^2}{16\Delta^2\Delta\omega_q}, \tag{2.5}$$

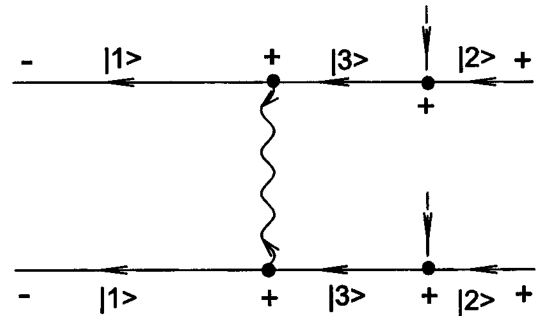
where Ω_R and $|\Omega'|$ are the Rabi frequencies for the classical and quantum modes, respectively. The latter is defined as $(|\Omega'|/2)^2=|d_{13}|^2\langle\epsilon^\dagger\epsilon\rangle/\hbar^2$, where d_{13} is the transition dipole moment for $|1\rangle\rightarrow|3\rangle$, and this Rabi frequency is proportional to the simultaneous normal-type amplitude correlations of the quantum mode $\langle\epsilon^\dagger\epsilon\rangle$, where ϵ and $\epsilon^\dagger$ are the annihilation and creation field operators, respectively. The spectral parameters contributing to the rate constant are the optical detuning of the interacting fields from atomic resonance Δ (see Fig. 1) and the spectral bandwidth of the quantum mode $\Delta\omega_q$. The subscripts in the collective states of the modified density operator (2.4) indicate which atom underwent the Raman transition. It is important for the expansion (2.4), written using the assumptions of perturbation theory, that $NRt\ll 1$, i.e., there is only a small probability for even one atom of the ensemble to make a transition to state $|2\rangle$.

Let us assume now that the quantum field is a realization of the spontaneous parametric down-conversion (SPDC) process generating weak quantum light far below threshold. In this case, due to anomalous amplitude correlations existing in the quantum field, there is an additional set of diagrams contributing to transformations of the double-particle density matrix as well as of the collective density matrix of the whole ensemble:



(2.6)

and



(2.7)

In these graphs the wavy lines are associated with anomalous-type Green functions of the quantum light.

Both the graphs (2.6) and (2.7) describe double-particle coherent correlations between the $|1\rangle$ and $|2\rangle$ Zeeman sublevels in the ground states of atoms. Taking into account all sets of diagrams (2.6) and (2.7) one obtains the following contribution to the collective density operator:

$$\hat{\rho}''(t)=\chi t\sum_{i>j=1}^{N}|1,1,\ldots,2_i,\ldots,2_j,\ldots,1\rangle\langle 1,\ldots,1|+\chi^* t\sum_{i>j=1}^{N}|1,\ldots,1\rangle\langle 1,1,\ldots,2_i,\ldots,2_j,\ldots,1|, \tag{2.8}$$

where χ and χ^* are the coupling constant and its complex conjugate associated with the amplitude of cooperative scattering shown by diagrams (2.6) and (2.7).

$$\chi\sim\frac{\Omega_R^2|\tilde{\Omega}'|^2}{16\Delta^2\Delta\omega_q}e^{2i(\theta-\theta_0)}, \tag{2.9}$$

where the Rabi frequency $(|\tilde{\Omega}'|/2)^2=|d_{13}|^2|\langle\epsilon\epsilon\rangle|/\hbar^2$ is determined by the simultaneous anomalous-type correlation function of the quantum mode $\langle\epsilon\epsilon\rangle=|\langle\epsilon\epsilon\rangle|e^{2i\theta}$. The phase θ_0 relates to the complex amplitude of the classical mode. In Eq. (2.9) we assumed that the spectral distribution of the anomalous-type amplitude correlations has the same spectral scale $\Delta\omega_q$ as the normal-type correlation function.

The main feature of the modification of the density operator by such a cooperative process is that it leaves the system in a pure state. To show this, let us rewrite the total density operator given by the sum of $\hat{\rho}'(t)$ and $\hat{\rho}''(t)$ in the following form:

$$\hat{\rho}(t)=\hat{\rho}'(t)+\hat{\rho}''(t)=(1-NRt)|\Psi_{Sq}^{(2)}(t)\rangle\langle\Psi_{Sq}^{(2)}(t)|+Rt\sum_{i,j=1}^{N}|1,1,\ldots,2_i,\ldots,1\rangle\langle 1,1,\ldots,2_j,\ldots,1|, \tag{2.10}$$

where $|\Psi_{Sq}^{(2)}(t)\rangle$ is the second-order approximation over the quantum field amplitude to the wave function $|\Psi_{Sq}(t)\rangle$, which can be generated from the initial state by the squeezing-type evolutionary operator

$$|\Psi_{Sq}(t)\rangle=\exp\left(-\frac{i}{\hbar}\hat{H}_{eff}t\right)|1,1,\ldots,1\rangle \qquad (2.11)$$

driven by the effective Hamiltonian

$$\hat{H}_{eff}=\sum_{i>j=1}^{N} i\hbar\chi\tau_{+}^{(i)}\tau_{+}^{(j)}-i\hbar\chi^{*}\tau_{-}^{(i)}\tau_{-}^{(j)}. \qquad (2.12)$$

The spin projectors $\tau_{\pm}^{(i)}$ are given by

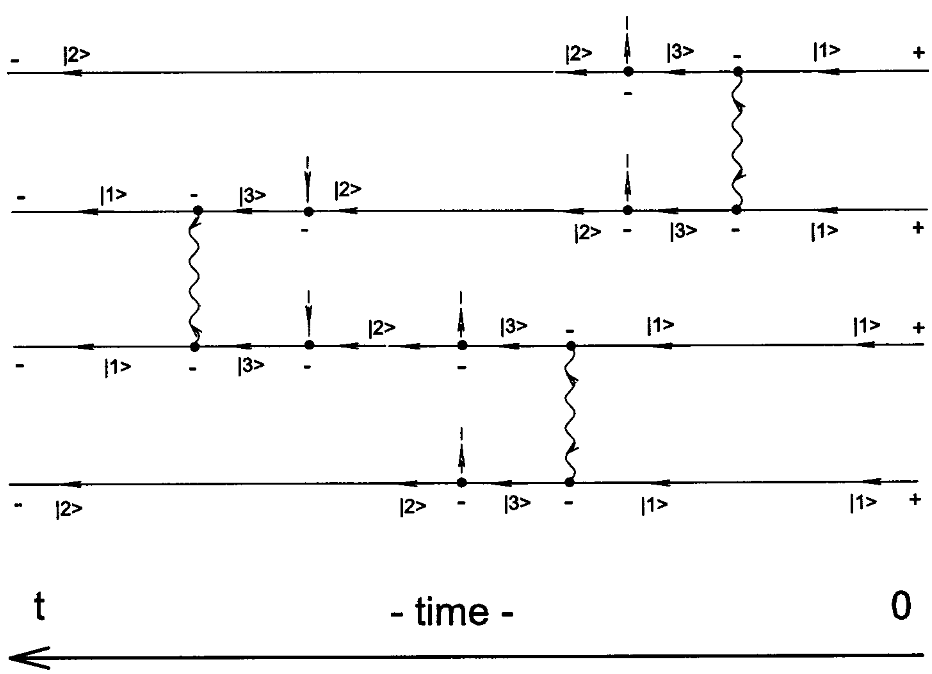
$$\tau_{+}^{(i)}=|2\rangle\langle 1|_{i}, \quad \tau_{-}^{(i)}=|1\rangle\langle 2|_{i} \qquad (2.13)$$

for each ith atom of the ensemble. Let us recall here that Eq. (2.10) is written under the assumptions that $NRt\ll 1$ and $N|\chi|t\ll 1$.

The definitions of the rate constant of the independent Raman-type transition R, given by Eq. (2.5), and the cooperative coupling constant χ, given by Eq. (2.9), basically differ in their dependence on the amplitude fluctuations associated with the quantum mode. The quantum mode Rabi frequencies which contribute to them depend on the normal-type correlation function $\langle\epsilon^{\dagger}\epsilon\rangle$ in the case of R and on the anomalous-type correlation function $\langle\epsilon\epsilon\rangle$ in the case of χ. For a SPDC process operating in the far-subthreshold regime, these correlation functions satisfy the inequality $|\langle\epsilon\epsilon\rangle|\gg\langle\epsilon^{\dagger}\epsilon\rangle$. Then, as follows from Eq. (2.10), if the probability of spin flips associated with the normal-type independent Raman transitions becomes negligible compared with the dynamical cooperative evolution, as long as the inequality $R\ll|\chi|$ is satisfied, the modified density operator describes a pure collective state slightly corrected by the effect of spin squeezing in the lowest nonvanishing order.

The excitation conditions leaving the system in a pure but squeezed state are satisfied if the ensemble is illuminated by a rare flux of pairs of strongly correlated photons created in a far-subthreshold SPDC process. Then the rate of independent Raman transitions R becomes smaller as the degeneracy parameter, i.e., the average number of photons in the coherence volume of the quantum field, becomes smaller. If the inequality $R\ll|\chi|$ is satisfied, it is possible to make the interaction time longer up to conditions when $N|\chi|t>1$ but still $NRt\ll 1$. Then the "coarse-grained" time-averaged dynamics of the spin subsystem is scaled by the time $1/N|\chi|$, which becomes quite short for large N. The self-consistency of our approach is restricted by the validity of the Markovian approximation in avoiding an overlap of different excitation processes during the shortest correlation time $\tau_c\sim\Delta\omega_q^{-1}$ and averaging the dynamics of the atomic subsystem over a time interval much longer than τ_c. This is legitimated by the inequality $N|\chi|\ll\Delta\omega_q$, which gives us an important restriction on the number of participating atoms.

If $N|\chi|t>1$ the dynamical evolution described by diagrams (2.6) and (2.7) should be extended to higher orders and, in accordance with Eq. (2.11), should generate a spin squeezed state. Indeed, the effective Hamiltonian (2.12) has order $\hbar N|\chi|$ and for the duration of an interaction process such as $N|\chi|t>1$, the higher-order corrections to the ladder-type diagrams (2.6) and (2.7), which couple not only one pair but many pairs of atoms, should be included in the perturbation theory expansion. An example of a diagram showing the contribution of higher orders in the case of four particles is given by



(2.14)

The full set of diagrams of such type can be generated by expansion of the evolutionary operator (2.11). It is crucial that, for durations long enough that the contributions of higher orders in expansion of the evolutionary operator become important, the independent Raman scattering still can satisfy the conditions of the perturbation theory approach at first order. In reality, the interaction time t is restricted in its upper limit by the relaxation time Γ^{-1}, which is associated with spontaneous Rayleigh-Raman scattering of the classical mode and describes relaxation of the coherence induced in the system (see the discussion in Ref. [10]). In this case, the mechanism of cooperative and correlated Raman scattering can lead to effective squeezing of the collective spin variables of the ensemble only if $N|\chi|/\Gamma \gg 1$. This inequality can be overcome for weak quantum light only for a large number of atoms N, which, in the typical conditions of a magneto-optic trap, can be increased up to 10^9.

It may seem unusual that illumination of a coherent atomic ensemble by weak quantum light is proposed here as a mechanism for effective squeezing of its spin subsystem. Let us point out here that in an ideal situation of an ultracold and isolated atomic ensemble all the internal relaxation processes become suppressed and even negligible. Then the result of each event of cooperative interaction of the correlated photon pair and the classical mode with the atomic sample, described by the dynamical Hamiltonian evolution, can be accumulated and stored by the system for a time less than Γ^{-1}. Physically, to satisfy the conditions discussed above, cooperative swapping between the quantum and classical modes should be achieved after many photon pairs have been scattered by the medium but at the same time not one photon was scattered independently. In this context ''many photon pairs'' means a number greater than unity but still comparable with unity, and ''no-one'' means that the degeneracy parameter is so small that the statistical probability of realizing any independent Raman scattering during the interaction cycle is negligible. In such conditions the initial spin wave function will be essentially modified and transformed to the wave function of a squeezed spin state. It is a basic property of the process discussed that for large N the dynamical evolution transforms the atomic system to the squeezed state before other non-correlated processes turn it into a mixed state. For methodical purity of our discussion, let us mention here again that for the validity of our approach in the case of a macroscopic number of atoms N, the amplitude of the cooperative process should be restricted by the inequality $N|\chi| < \Delta\omega_q$, which can actually be important if N is very large.

III. HEISENBERG-LANGEVIN APPROACH: DYNAMICS OF COLLECTIVE VARIABLES

Let us define the following collective quasispin operators of the ensemble:

$$\hat{T}_1=\sum_{j=1}^{N}\frac{1}{2}(|2\rangle\langle 1|_j+|1\rangle\langle 2|_j),$$

$$\hat{T}_2=\sum_{j=1}^{N}\frac{1}{2i}(|2\rangle\langle 1|_j-|1\rangle\langle 2|_j),$$

$$\hat{T}_3=\sum_{j=1}^{N}\frac{1}{2}(|2\rangle\langle 2|_j-|1\rangle\langle 1|_j), \tag{3.1}$$

which obey the commutation relations of the vector components of the angular momentum

$$[\hat{T}_\alpha,\hat{T}_\beta]=i\varepsilon_{\alpha\beta\gamma}\hat{T}_\gamma \tag{3.2}$$

for $\alpha\neq\beta\neq\gamma$, each ranging from 1 to 3, and $\varepsilon_{\alpha\beta\gamma}=\pm 1$ depending on the order of indices [15]. The dynamics of collective variables can be described in the Heisenberg-Langevin formalism. In this approach the dynamical evolution of the operators driven by the Hamiltonian (2.12) interferes with damping terms caused by dissipation processes. Among these, the most important are the processes of independent stimulated Raman scattering of the quantum mode and of incoherent Rayleigh-Raman scattering of the classical mode. The coherent scattering of the classical mode is a dynamical process manifested in shifting of the sublevel $|2\rangle$. We assume that this light shift can be compensated by an additional external magnetic field and ignore it in further analysis. We also ignore the damping terms associated with very weak processes of incoherent scattering of the quantum mode.

With these assumptions the Heisenberg-Langevin equations are given by

$$\dot{\hat{T}}_3=2\chi[\hat{T}_1^2(t)-\hat{T}_2^2(t)]-(\Gamma_{21}+R)\left[\hat{T}_3(t)+\frac{N}{2}\frac{\Gamma_{21}-R}{\Gamma_{21}+R}\right]+\hat{L}_3(t),$$

$$\dot{\hat{T}}_1=-\chi\{\hat{T}_1(t),\hat{T}_3(t)\}_+-\frac{\Gamma+R}{2}\hat{T}_1(t)+\hat{L}_1(t),$$

$$\dot{\hat{T}}_2=\chi\{\hat{T}_2(t),\hat{T}_3(t)\}_+-\frac{\Gamma+R}{2}\hat{T}_2(t)+\hat{L}_2(t). \tag{3.3}$$

Here, for the sake of simplicity, we assumed the coupling constant χ of the effective Hamiltonian to be real and positive. By $\{\cdots\}_+$ we denoted the anticommutator of operators. The first terms on the right-hand sides are the commutators of the effective Hamiltonian (2.12) with the operators of each collective variable (3.1). The second terms on the right-hand sides are normal kinetic-type terms, which can be obtained from the corresponding relaxation terms written as a fragment of a kinetic equation for the single-particle density matrix. These terms are responsible for the relaxation processes tending to convert the system to an equilibrium state. Such a state, which should not actually be realized just because of the dynamical terms, is related to the equilibrium conditions obtained by Poissonian-type noncorrelated processes of independent Raman scattering of the quantum mode and of spontaneous Rayleigh-Raman scattering of the classical mode. The important parameter in Eqs. (4.3) is the rate of decoherency Γ, which can be expanded as the sum $\Gamma=\Gamma_{22}+\Gamma_{21}$, where Γ_{22} and Γ_{21} are, respectively, the rates of Rayleigh- and Raman-type transitions caused by incoherent scattering of the classical mode. The last terms on the right-hand sides of Eqs. (3.3) are the Langevin forces.

In spite of the clear structure of Eqs. (3.3), they are not so easy to discuss. We do not actually know the Langevin forces $L_\alpha(t)$ at an arbitrary moment of time t, when, in the Schrödinger picture, the collective density operator is driven by the high orders of the perturbation theory expansion. Moreover, the nonlinear structure of the dynamical terms makes it more difficult to analyze these equations in the steady state regime. But in the initial stage of evolution, when there is no depletion of the spin orientation $\overline{T_3}(t)\rightarrow -N/2$, the last equation can be simplified by substituting the Heisenberg operator $\hat{T}_3(t)$ by its average value. Actually, this assumption is satisfied if the excitation time is sufficiently short (see the discussion in the previous section), and it leads us to a quasistationary solution where the Langevin forces can be described by their correlation properties:

$$\langle \hat{L}_2(t')\hat{L}_2(t)\rangle = D_{22}\delta(t'-t),$$

$$D_{22}=(\Gamma+R)\frac{N}{4}+\frac{N(N-1)}{2}R. \tag{3.4}$$

The diffusion coefficient D_{22} can be found by direct comparison of the iterative solution of Eqs. (3.3) with the parallel evaluation based on expansion of the density matrix (2.10). Then the quasistationary variance of the T_2 component is given by

$$\langle \hat{T}_2^2\rangle = \frac{(\Gamma+R)/2+(N-1)R}{\chi N+(\Gamma+R)/2}\,\frac{N}{4}\rightarrow \frac{R}{\chi}\,\frac{N}{4}, \tag{3.5}$$

where the interaction time is restricted by the inequality $(\chi N)^{-1}\ll t\ll \Gamma^{-1}$. The right-hand side shows the lowest physical limit available for this mechanism of spin squeezing where the standard quantum variance $N/4$ can be suppressed by the factor $R/\chi\ll 1$.

The observation of squeezing in a system of quasispin components needs some special discussion. Because of the tensor nature of the alignment components, the squared variance of the quasispin is distributed in normal coordinate space not in an elliptical but in a rosette-type area in the x,y plane, as shown in Fig. 2. In this figure the distributions of the squared variance for a general type of spin squeezed state and for a quasispin squeezed state are compared. There are two different sets of orthogonal directions associated with a probe of either T_1 or T_2 quasispin (alignment) components. The squeezing of the T_2 component and antisqueezing of the T_1 component of the quasispin can be observed in a special interference detection scheme generalizing the idea of Faraday detection of the normal spin squeezed states. To show this let us introduce first the following basis set as an alternative to the basis of Zeeman states (see [14]):

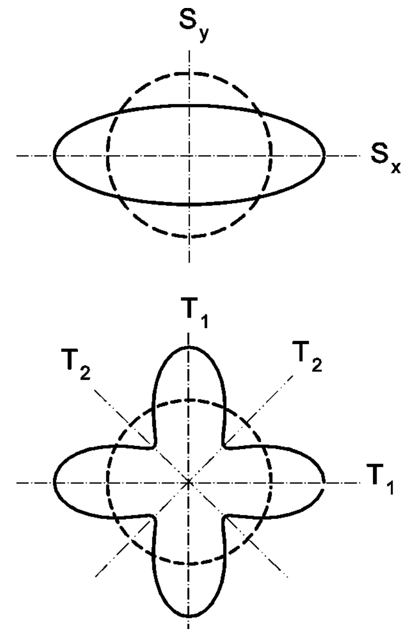


FIG. 2. The uncertainty distributions in coordinate space for a general type of spin squeezing between S_x and S_y fluctuations and for quasispin squeezing between T_1 and T_2 fluctuations (see text).

$$|x\rangle=\frac{1}{\sqrt{2}}(|1,-1\rangle-|1,+1\rangle)=\frac{1}{\sqrt{2}}(|1\rangle-|2\rangle),$$

$$|y\rangle=\frac{i}{\sqrt{2}}(|1,-1\rangle+|1,+1\rangle)=\frac{i}{\sqrt{2}}(|1\rangle+|2\rangle),$$

$$|z\rangle=|1,0\rangle. \tag{3.6}$$

The same basis can be defined for the upper level of the $F=1\rightarrow f=1$ transition. As can be straightforwardly verified, linearly polarized light along the x direction selects transitions only between $|y\rangle$ lower and $|z\rangle$ upper states. Recall that the $|z\rangle$ sublevel in the ground state is not populated. In turn, a probe by light polarized along the y direction selects the transitions between $|x\rangle$ lower and $|z\rangle$ upper states.

Consider now the Mach-Zehnder-type interferometer shown in Fig. 3. The plane of the interferometer is orthogonal to the quantization direction z. The input and output mirrors are polarization insensitive 50-50 beam splitters, and the phases associated with the optical paths in the interferometer arms are additionally shifted by $\pi/2$. Let us assume that linearly polarized classical light comes into one port of the interferometer and splits in such a way that two beams cross the atomic cloud in orthogonal directions (see Fig. 2). Then the beam propagating along the y direction will be linearly polarized along the x direction and the beam propagating along the x direction will be correspondingly polarized along the y direction. Both the beams will be coherently scattered in the forward direction, preserving their polarizations. Then on measuring the imbalance in the photocurrents in detectors D_1 and D_2, the average signal will be proportional to the expectation value of the observable:

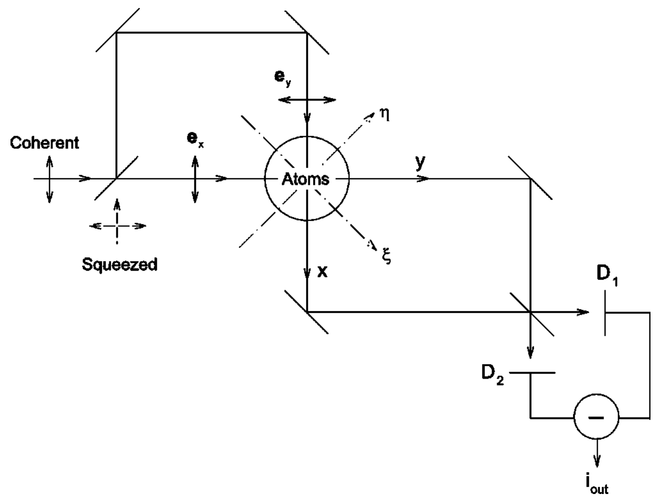


FIG. 3. Scheme of an interferometer for detection of squeezing or antisqueezing of the collective atomic quasispin. Linearly polarized coherent light comes in to one of the interferometer ports. The output photocurrent i_{out} of the balanced detector reveals the phase mismatch between the field amplitudes with $\mathbf{e}_x$ and $\mathbf{e}_y$ polarization coherently scattered by the atomic ensemble, and the mismatching is associated with the T_1 quasispin component (see text). The sensitivity of the scheme can be increased if a portion of the squeezed light comes in additionally to the second port of the interferometer. The squeezed T_2 component can be detected if the light beams cross the atomic ensemble along the ξ and η directions rotated at an angle $\pi/4$.

$$\sum_{j=1}^{N}\frac{1}{2}(|y\rangle\langle y|_j-|x\rangle\langle x|_j)=\hat{T}_1, \tag{3.7}$$

where the sum is extended over all atoms of the ensemble. This basic statement is valid if the difference in the phase shifts associated with coherent forward scattering of the light polarized along either the x or y directions is small enough. Actually, for a transparent medium this difference will always be small as long as it exists only as a fluctuation. Thus measuring the photocurrent spectrum in the low-frequency domain gives us information about the variance of T_1. The sensitivity of the scheme can be increased if a portion of the squeezed light additionally comes into the second port of the interferometer (see Fig. 2). To observe the T_2 component, instead of a combination of the x and y axes, the basis of linear polarizations should be used, as well as the basis of atomic states referred to the ξ,η axes rotated at an angle $\pi/4$, which is a common idea in measuring the alignment tensor (see Fig. 2 and the review [16] for more detail). It can be straightforwardly shown that in this frame

$$\sum_{j=1}^{N}\frac{1}{2}(|\eta\rangle\langle\eta|_j-|\xi\rangle\langle\xi|_j)=\hat{T}_2, \tag{3.8}$$

where the basis of $|\xi\rangle$ and $|\eta\rangle$ states can be obtained from the basis of $|x\rangle$ and $|y\rangle$ states by rotational transformation by an angle $\pi/4$ around the z axis.

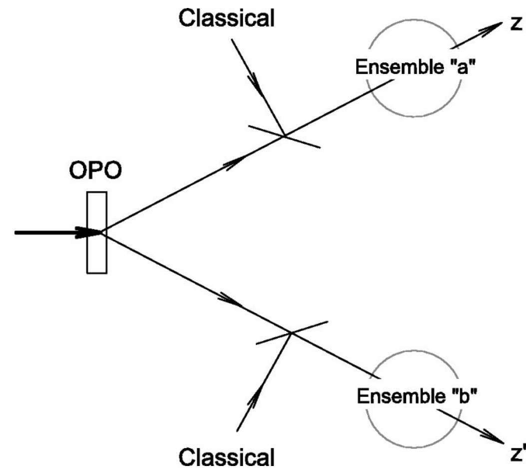


FIG. 4. The proposed scheme of optical coupling for entangling the spin subsystems of two spatially separated ensembles. The correlated photons created by an OPO source come in to different ensembles a and b. After mixing with classical light in orthogonal polarization they initiate a Λ-type excitation between the atoms of different ensembles similar the one shown in Fig. 1.

IV. ENTANGLED QUASISPIN STATES

A similar process can be utilized to create entanglement in the spin subsystems of two spatially separated atomic ensembles, which we denote a and b. For this goal, one photon of the correlated pair should be sent to one ensemble and the second photon to the other ensemble (see Fig. 4). Then the process of cooperative and correlated scattering described by diagrams (2.6) and (2.7) will occur between the atoms from different atomic ensembles. The dynamical evolution of the quasispin components will be driven by the following effective Hamiltonian:

$$\hat{H}_{eff}=\sum_{i=1}^{N_a}\sum_{j=1}^{N_b} i\hbar\chi\tau_+^{(i)}\tau_+^{(j)}-i\hbar\chi^*\tau_-^{(i)}\tau_-^{(j)}, \tag{4.1}$$

where the projection operators $\tau_\pm^{(i)}$ and $\tau_\pm^{(j)}$ under the sums relate to the atoms of the different ensembles. We assume the atomic clouds to be macroscopically identical with the same number of atoms $N_a=N_b=N$. The cooperative transitions described by the Hamiltonian (4.1) can be initiated, for example, by the scattering of the photon pairs created by a subthreshold optical parametric oscillator (OPO) based on a SPDC process of type I with noncollinear phase matching conditions. Then the classical modes in orthogonal polarizations can be admixed with the aid of polarization sensitive beam splitters inserted near the atomic clouds. The definitions of the Zeeman states and of the quasispin components connected with the propagation directions of the light beams and with the directions of their polarizations should be referred to the local frame near each cloud.

Based on arguments similar to those of the previous sections, it can be shown that after a short period of optical excitation the collective quasispin components become entangled in such a way that

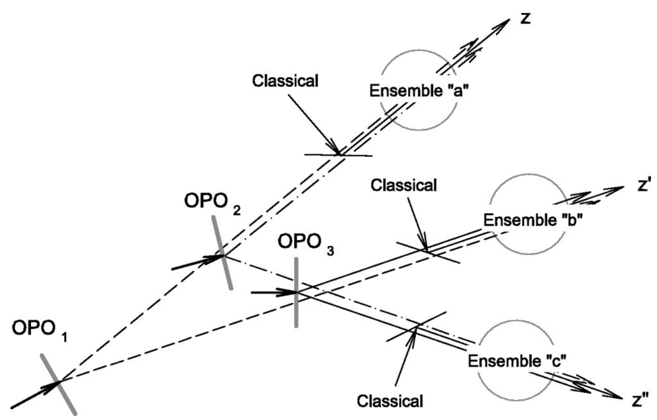


FIG. 5. The scheme of optical coupling for entangling the spin subsystems of three spatially separated ensembles. The radiation of three weak OPO sources mixed with classical modes in orthogonal polarizations initiates the Λ-type excitation between the atoms of different ensembles similar to that shown in Fig. 1.

$$\langle [T_1^{(a)} - T_1^{(b)}]^2 \rangle < \frac{N}{2},$$

$$\langle [T_2^{(a)} + T_2^{(b)}]^2 \rangle < \frac{N}{2}. \tag{4.2}$$

This type of quantum correlation of the quasispin components $T_{1,2}^{(a)}$ and $T_{1,2}^{(b)}$, defined, respectively, for the a and b ensembles, occurs for the coupling constant χ being real and positive. Here, as in the case of squeezing, the entangling mechanism should work as long as the independent Raman-type transitions are negligible. Actually, the standard variance for the observables on the left-hand side can be suppressed by a factor of order R/χ. The observation of entanglement in the spin subsystems of two atomic ensembles can be done with an interferometer similar to the one suggested in the previous section for observation of spin squeezing in one cloud.

The idea can be further generalized, and in Fig. 5 is shown a possible scheme to generate an entangled state in the spin subsystems of three atomic ensembles numbered a, b, and c. Three OPO crystals generate the photon pairs in a SPDC process of type I in the far-subthreshold regime. As long as the efficiency of the SPDC process is weak, each crystal is transparent to the radiation outgoing from another crystal. This means that nonlinear interaction between the photons created in different crystals is negligible. Thus, it might be possible to organize that two collinear beams of the radiation coming from two different crystals can always cross each ensemble, as is shown in Fig. 4. The classical modes in orthogonal polarizations can be admixed in beam splitters inserted just before the atomic clouds. For the case of the optical transition $F=1 \rightarrow f=1$ and for the Λ-type excitation realized in circular polarizations, the directions of the z axes in the local reference frames near each cloud should be associated with the directions of the light beams.

Ideally, the process will be driven by the following effective Hamiltonian:

$$\hat{H}_{eff} = \hat{H}_{ab} + \hat{H}_{ac} + \hat{H}_{bc}, \tag{4.3}$$

where each term on the right-hand side is given by expression (4.1) and the sum extends over all possible combinations of the ensemble pairs. This introduces quantum correlations in the spin subsystems and leads to the following entanglement of the quasispin components of all three ensembles:

$$\langle [T_1^{(a)} - T_1^{(b)}]^2 \rangle < \frac{N}{2},$$

$$\langle [T_1^{(a)} - T_1^{(c)}]^2 \rangle < \frac{N}{2},$$

$$\langle [T_1^{(b)} - T_1^{(c)}]^2 \rangle < \frac{N}{2},$$

$$\langle [T_2^{(a)} + T_2^{(b)} + T_2^{(c)}]^2 \rangle < \frac{3N}{4}. \tag{4.4}$$

This type of macroscopic entanglement relating to a system of three harmonic oscillators is analogous to the so called Greenberger-Horne-Zeilinger (GHZ) entanglement normally defined in the set of discrete variables (see the review [17]). We maintain here the terminology of discrete variables to emphasize the similar meaning of this type of entanglement in the context of quantum information protocols based on continuous variables as in the normal GHZ states in discrete variable protocols. But let us point out that, rigorously speaking, such states possessing strong correlation in terms of collective variables at the same time have only weak interparticle entanglement. In other words, by taking one or a few atoms away from the entire system we do not essentially demolish its macroscopic correlation.

It is quite easy to introduce further possible generalizations of spin subsystems containing of many ensembles. The scheme for the interferometer described in the previous section for squeezing detection can also be generalized to entanglement observation. However, it is quite clear that technical difficulties would often arise if the number of ensembles participating in the process were great enough. This problem seems general in any possible proposals for entanglement in a system of continuous variables.

V. CONCLUSION

In this paper we proposed an optical mechanism for spin squeezing or entanglement, which can be realized in an ensemble or ensembles of cold alkali-metal atoms. The mechanism utilizes the idea of optical pumping and is based on the process of stimulated cooperative Raman-type scattering of the correlated photon pairs generated in spontaneous parametric down-conversion of type I. We showed how the squeezed and different types of entangled states, as candidates for the basic logic elements in quantum information protocols based on continuous variables, could be prepared in a system of spin-made harmonic oscillators. The example of an optical transition where the angular momenta of both the lower and upper states are equal to 1, is not a serious restriction and was chosen for our discussion mainly because of its methodical clarity. The proposed scheme of quasispin squeezing should work in the more general situation of arbitrary closed hyperfine transitions or in the multiplet of several transitions. That is because it is crucial only to organize the process of cooperative Raman scattering of correlated photons during the short time of optical excitation of the originally coherent spin state. In such a general situation the squeezing or entanglement of the quasispin components would be properly described in terms not only of orientation and alignment but of the higher irreducible components as well. Let us point out here that for the proposed as well as for other possible schemes quasispin squeezing, special attention should always be paid to the detection channel. The interferometer scheme we considered is an illustration of this; it is based on the same idea as the common Faraday-type detection channel (see Ref. [11]), but it is different in its technical realization. In conclusion, let us also point out that the proposed scheme of spin squeezing or entanglement has an important advantage, particularly in the case of cold atomic ensembles, since it gives only negligible acceleration effects to atoms in a trap because the optical perturbation of the system is quite weak.

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