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## Autler–Townes Effect in Hyperfine Structure of Alkali-Atom $D_1$ line

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**Abstract**—The Autler–Townes effect is considered for the case where a strong control field acts in the frequency profile of a hyperfine transition taking into account the hyperfine interaction. An exact expression is derived for the susceptibility of the medium with respect to a probe field that acts in the frequency profile of a coupled transition of the alkali-atom  $D_1$  line. It is shown that the hyperfine interaction in the excited state may substantially modify the susceptibility of the medium. In particular, it is shown that, when the control field is tuned to wings of atomic transitions, the structure of the Autler–Townes resonances appears to be noticeably differing from what is predicted by the  $\Lambda$ -scheme approximation. It is found that tuning of the control field frequency to between hyperfine components of the excited state enhances the resonant structure in this region. The results are discussed in terms of the quantum memory protocols based on the effects of Raman scattering and electromagnetically induced transparency.

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### INTRODUCTION

A great number of optical phenomena important for nonlinear high-resolution spectroscopy are observed upon the interaction of several light fields that are resonant or nearly resonant to coupled transitions [1, 2]. In recent years, the interest in this problem has substantially arisen due to the development of quantum memory systems for the probe light pulse whose state is considered to be the information carrier [3–15]. In practical designs of the quantum memory protocols, three mechanisms are usually used, i.e., the photon-echo effect, the electromagnetically induced transparency (EIT), and the stimulated Raman scattering. All three mechanisms are based on the  $\Lambda$ -configured interaction at coupled transitions. To properly describe them, one must correctly calculate the lineshape for the interaction of the probe light with the atomic system in a strong field of the control light. This problem was primarily considered in [1] for the system of two states dressed by the interaction with the control mode and interrogated by the probe field acting in the frequency profile of a coupled transition. The authors of [1] called it “the Stark effect in rapidly varying fields.” At present, this process is more often referred to as the Autler–Townes effect. This paper is devoted to calculating the Autler–Townes resonance lineshape in the hyperfine structure of the alkali-atom  $D_1$  line, which is of certain interest for the development of practical recommendations for quantum memory systems in optically dense atomic media.

### SUSCEPTIBILITY OF MEDIUM THAT INTERACTS WITH CONTROL AND PROBE FIELDS

Figure 1 shows the considered energy-level structure of an alkali atom, which is exemplified by the  $^{133}\text{Cs}$  atom, that is subjected to the action of a strong monochromatic field acting at the frequency  $\omega$  of the  $D_1$  transition and that is interrogated by a weak probe light acting at the frequency of a coupled transition. It is assumed that only one Zeeman sublevel ( $F = 4, M = 4$ ) of the ground state, which we denote as  $|m\rangle$ , is populated. As can be seen from the assumed scheme of the excitation by the strong field in the right-handed polarization ( $\sigma_+$ ), the interaction of the populated state with the control mode proves to be completely suppressed. The strong field acts only upon the unpopulated Zeeman sublevel  $F = 4, M = 2$  (the state  $|m'\rangle$ ) and two Zeeman sublevels  $F = 3, M = 3$  (the state  $|n\rangle$ ) and  $F = 4, M = 3$  (the state  $|n'\rangle$ ) of the excited state.

The presence of the strong mode substantially affects the energy structure of the atom giving rise to a shift of the atomic levels and to the distortion of the absorption spectrum in the vicinity of the states  $|n\rangle$  and  $|n'\rangle$ , which is shown in Fig. 1 by thick dashed lines. In addition, there arises an additional resonance located in the vicinity of the frequency of the control field. If the strong mode is tuned to resonance with some atomic transition, then, for symmetry reasons, the strict separation of the atomic resonance becomes impossible and it splits into two, close in shape, quasi-energy sublevels. The existence of this resonant structure in the atom + control field combined system is revealed upon probing the medium by a weak field of a

left-handed circular polarization ( $\sigma_-$ ) acting at the frequency of the adjacent transition from the populated sublevel, as is shown in Fig. 1. This scheme corresponds to the standard method of observation of the Autler–Townes resonant structure [1, 2].

Let us separate the slowly varying amplitude in the positive-frequency component of the probe field

$$\mathcal{E}_{\text{Left}}^{(+)}(\mathbf{r}, t) = \epsilon(\mathbf{r}_{\perp}, z; t) e^{-i\bar{\omega}t + i\bar{k}z}, \quad (1)$$

This implies that a quasi-monochromatic wave with the carrier frequency  $\bar{\omega}$  and the wavenumber  $\bar{k} = \bar{\omega}/c$  propagates along the  $z$ -direction and that the diffraction divergence of the transverse profile of this wave, which is characterized by the coordinate  $\mathbf{r}_{\perp}$ , is ignored. By finding the Fourier component

$$\epsilon(\mathbf{r}_{\perp}, z; \Omega) = \int_{-\infty}^{\infty} dt e^{i\Omega t} \epsilon(\mathbf{r}_{\perp}, z; t) \quad (2)$$

under steady-state excitation by the control field, we can represent the propagation of the probe wave in the medium by the Maxwell equation

$$\begin{aligned} & \left[ -i\frac{\Omega}{c} + \frac{\partial}{\partial z} \right] \epsilon(\mathbf{r}_{\perp}, z; \Omega) \\ & = 2\pi i \frac{\bar{\omega}}{c} \tilde{\chi}(\mathbf{r}_{\perp}, z; \Omega) \epsilon(\mathbf{r}_{\perp}, z; \Omega), \end{aligned} \quad (3)$$

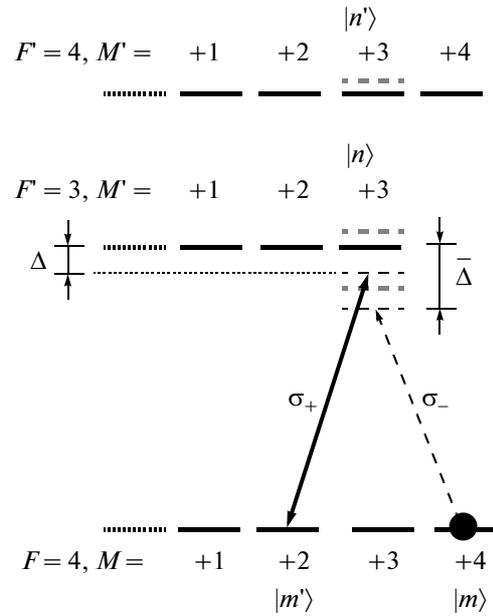
where we also separated the slow time dependence in the susceptibility of the medium so that, in the time domain,

$$\tilde{\chi}(\mathbf{r}_{\perp}, z; t-t') = e^{i\bar{\omega}(t-t')} \chi(\mathbf{r}_{\perp}, z; t-t'), \quad (4)$$

where  $\chi(\mathbf{r}_{\perp}, z; t-t')$  is the standard electrodynamic susceptibility, which describes the response of the polarization of the medium to the external probe field (1). When the medium is probed by a monochromatic field, it is natural to consider the spectral dependence of exactly this standard characteristic of the medium as a function of the frequency  $\bar{\omega}$  or of the detuning  $\bar{\Delta}$  from the atomic resonance (see Fig. 1)  $\chi = \chi(\dots; \bar{\omega}) = \chi(\dots; \bar{\Delta})$ . This dependence can be easily extracted if the function  $\tilde{\chi}(\dots; \Omega)$  is taken at  $\Omega = 0$ .

The susceptibility of the medium can be represented in the form of the following expansion:

$$\begin{aligned} \tilde{\chi}(\mathbf{r}_{\perp}, z; \Omega) = & - \sum_{n_1 = n, n'} \sum_{n_2 = n, n'} \frac{1}{\hbar} (\mathbf{d}\mathbf{e})_{n_1 m}^* (\mathbf{d}\mathbf{e})_{n_2 m} \\ & \times \int \frac{d^3 p}{(2\pi\hbar)^3} n_0(\mathbf{r}_{\perp}, z) f_0(\mathbf{p}) \\ & \times G_{n_1 n_2}^{(-)} \left( \mathbf{p}_{\perp}, p_z + \hbar\bar{k}; \hbar(\bar{\omega} + \Omega) + \frac{p^2}{2m} \right), \end{aligned} \quad (5)$$



**Fig. 1.** Energy diagram and assumed excitation scheme for the  $D_1$  line of the  $^{133}\text{Cs}$  atom. Atoms populate the upper hyperfine level of the ground state characterized by the greatest value of the projection of the angular momentum. The system is excited by a strong mode of a control field in the right-hand circular polarization ( $\sigma_+$ ) with the detuning  $\Delta$  and is interrogated by a weak mode of a probe beam in the left-handed ( $\sigma_-$ ) polarization in the frequency profile of a coupled transition. By probing the medium with a weak beam and varying the detuning frequency, one can observe the resonance structure in the excitation spectrum of the atom dressed by interactions with the control mode, i.e., the Autler–Townes effect. Positions of the resonant quasi-energy states are shown by bold dashed lines.

where we introduced the matrix elements of the projection of the dipole moment operator  $\mathbf{d}$  of the atom onto the unit vector  $\mathbf{e}$  of the polarization of the probe beam; the spatial distribution of atoms is described by their local concentration  $n_0(\mathbf{r}_{\perp}, z)$ , while the momentum distribution of the function  $f_0(\mathbf{p})$  is normalized by the condition

$$\int \frac{d^3 p}{(2\pi\hbar)^3} f_0(\mathbf{p}) = 1. \quad (6)$$

The main characteristic that controls the susceptibility of the medium is related to the set of the retarded Green's functions of the excited atomic state  $G_{n_1 n_2}^{(-)}(\mathbf{p}, E)$  that is contained in expansion (5) and that are dressed by the interaction with the strong control field. These functions correspond to all possible combinations of the subscripts  $n_1 = n, n'$  and  $n_2 = n, n'$  and are specified by their spatiotemporal Fourier expansion. The energy  $E = \hbar(\bar{\omega} + \Omega) + p^2/2m_0$  corresponds to the initial energy of the system atom + probe field, in which the energy of the state  $|m\rangle$  is taken to be zero.

The momentum of the excited atom is shifted with respect to its ground state by the transferred momentum of the probe mode photon  $p'_z = p_z + \hbar\bar{k}$ .

For the scheme of the transitions shown in Fig. 1, the Green's functions found by the diagrammatic methods [14] can be represented in the form

$$G_{nn}^{(--)}(\mathbf{p}, E) = \hbar \left\{ E - \frac{p^2}{2m_0} - E_n + i\hbar\frac{\gamma}{2} - \frac{|V_{nm'}|^2 \left[ E - \frac{p^2}{2m} - E_{n'} + i\hbar\frac{\gamma}{2} \right]}{[E - E_{n'+}(\mathbf{p}, \omega)][E - E_{n'-}(\mathbf{p}, \omega)]} \right\}^{-1}, \quad (7)$$

$$G_{n'n}^{(--)}(\mathbf{p}, E) = \frac{V_{n'm'}V_{nm'}^*}{[E - E_{n'+}(\mathbf{p}, \omega)][E - E_{n'-}(\mathbf{p}, \omega)]} \times G_{nn}^{(--)}(\mathbf{p}, E).$$

...

Two other functions can be obtained by the obvious permutation of the subscripts  $n \leftrightarrow n'$ . We introduced the following denotation:  $V_{nm'} = (\mathbf{dE})_{nm'}$ ,  $V_{n'm'} = (\mathbf{dE})_{n'm'}$  are the matrix elements of interaction with the control mode;  $\mathbf{E}$  is the complex amplitude of its positive-frequency component  $\mathbf{E}^{(+)}(\mathbf{r}, t) = \mathbf{E} \exp(-i\omega t + i\mathbf{k}\mathbf{r})$ ;  $E_n$ ,  $E_{n'}$  are the unperturbed energies of the excited states of the atom; and  $\gamma$  is the rate of the natural radiative decay of the excited state. The energy denominators of relations (7) include quasi-energies that correspond to the dressing of excited states by interactions with vacuum modes and the control field under the condition that the effect of the hyperfine interaction is neglected,

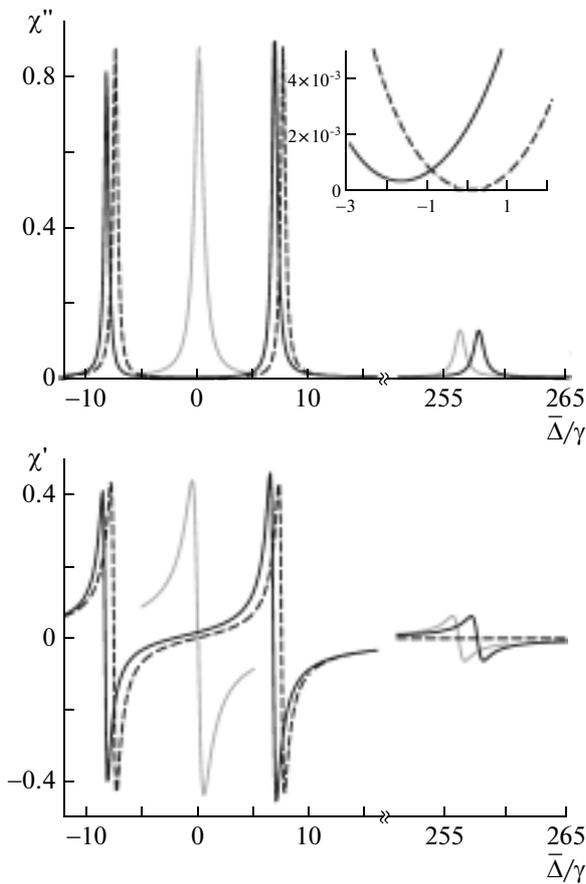
$$E_{n\pm}(\mathbf{p}, \omega) = E_m + \frac{p^2}{2m_0} + \frac{\hbar}{2} \left[ \omega - \frac{\mathbf{k}\mathbf{p}}{m_0} + \omega_{nm'} - i\frac{\gamma}{2} \right] \pm \left[ |V_{nm'}|^2 + \frac{\hbar^2}{4} \left( \omega_{nm'} - \omega + \frac{\mathbf{k}\mathbf{p}}{m_0} - i\frac{\gamma}{2} \right)^2 \right]^{1/2} \quad (8)$$

and, a similar expression can be written in the case of  $n'$ . Here,  $E_m$  is the energy of the lower state of the excited transition, which, for the system of the degenerate Zeeman sublevels of the lower state (Fig. 1), can be taken for the coordinate origin, so that  $E_m = E_{m'} = 0$ . The quasi-energies (8) are the intermediate parameters of the problem, which correspond to its solution when only one excited state (either  $|n\rangle$  or  $|n'\rangle$ ) is taken into account. The real position of the resonant poles is determined by the structure of medium susceptibility (5), which all Green's functions contribute to.

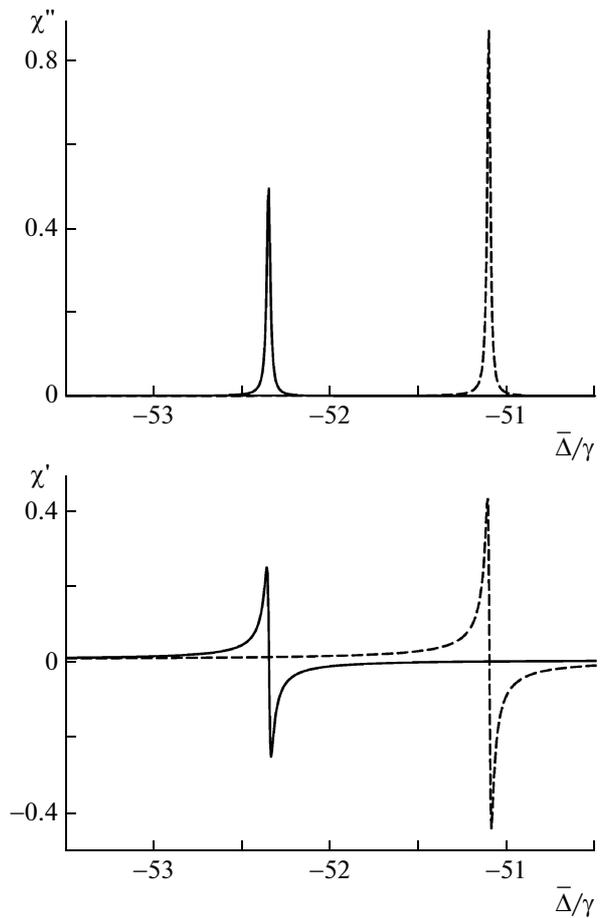
## RESULTS AND DISCUSSION

We numerically calculated the susceptibility  $\chi = \chi'(\bar{\Delta}) + i\chi''(\bar{\Delta})$  for the model of a homogeneous medium. The results are presented in units of the dimensionless density  $n_0(\lambda/2\pi)^3$  ( $n_0$  is the atomic concentration and  $\lambda$  is the light wavelength) for different values of detuning  $\Delta$  (Fig. 1). The atoms are assumed to be located in a magneto-optical trap under standard conditions (at a temperature below the Doppler limit but above the recoil limit), which allowed us to neglect the effects of atomic motion. In this calculation, the fact that the matrix elements  $V_{nm'}$  and  $V_{n'm'}$  cannot be considered to be independent parameters that are strongly related to one another by the scheme of summation of the electron and nuclear angular momenta with allowance for the hyperfine interaction in the excited state, is of fundamental importance. As the parameter that characterizes the strength of the interaction with the control mode, we determined the Rabi frequency defined by the matrix element that relates the control mode with the lower hyperfine sublevel of the excited state  $\Omega_c = 2|V_{nm'}|/\hbar$ . For the presented results, we performed a preliminary analysis using a rougher model approximation of the  $\Lambda$ -scheme, which formally corresponds to the existence of a single excited sublevel, namely, the state  $|n\rangle = |F = 3, M = 3\rangle$  for the case of the  $^{133}\text{Cs}$  atom.

Figures 2–4 show variations of the imaginary (absorptive) and real (dispersive) components of the medium susceptibility for the same values of the Rabi frequency  $\Omega_c = 15\gamma$  and different detuning of the control mode  $\Delta$ . In our case, there is no real absorption, and the imaginary component of the susceptibility is responsible for the losses related to the process of incoherent scattering. The dependence in Fig. 2 corresponds to the case of resonant interaction of the control mode with the transition  $|m'\rangle \rightarrow |n\rangle$ . For this case, we presented all the three components of the Autler–Townes triplet and showed the distinction from the model of two levels with no allowance for the  $|n'\rangle$  state. Visually, this distinction may seem insignificant. However, as was shown in the inset, the existence of the hyperfine interaction provides a qualitative effect. The ideal conditions for the existence of the EIT in the system disappear and the point of smallest absorption appears to be shifted towards lower frequencies. The value of the light-induced shift appears to be substantial, and this effect was observed in one of the first experiments on slow-light propagation in optically dense media [16]. From a practical point of view, however, to describe the probe light pulse delay related to the dispersion of the medium susceptibility, this distinction from the  $\Lambda$ -scheme approximation is insignificant. It is also noteworthy that, e.g., in the case of the EIT protocol used as a version of the quantum memory, the most favorable situation is when  $\Omega_c$



**Fig. 2.** Shape of absorptive and dispersive components of susceptibility of medium modified by action of strong field for the case of resonant excitation ( $\Delta = 0$ ) and Rabi frequency  $\Omega_c = 15\gamma$ . Solid curve corresponds to the hyperfine interaction taken into account and dashed curve corresponds to  $\Lambda$  approximation. Bold curve corresponds to the susceptibility profile unperturbed by the action of the control field. The inset shows that, due to the second hyperfine sublevel, the EIT resonance proves to be shifted to the red wing and the transparency ceases to be perfect.



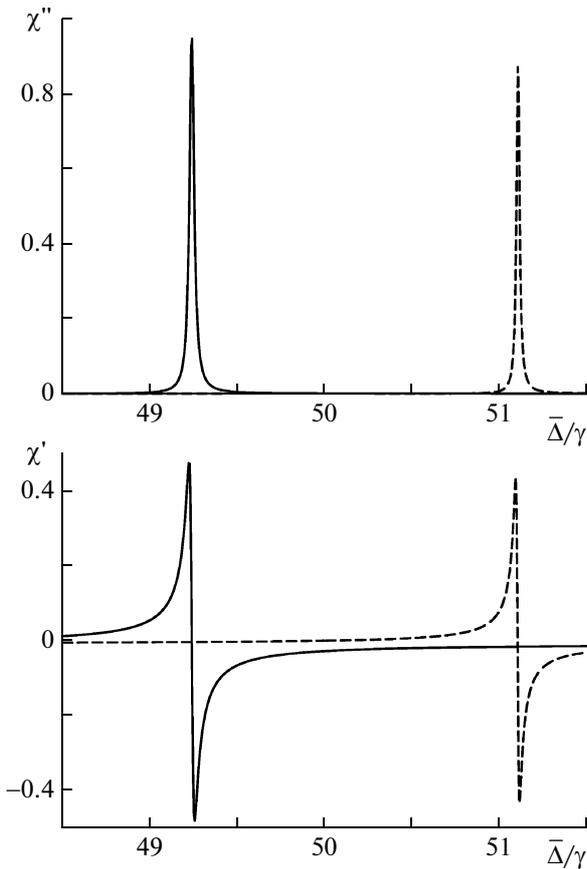
**Fig. 3.** Same as in Fig. 2, but for detuning of the control mode  $\Delta = -50\gamma$  and Rabi frequency  $\Omega_c = 15\gamma$ . The structure of Autler–Townes resonance is only shown in the vicinity of  $\bar{\Delta} \sim \Delta$ . The structure of other components of the triplet is close to the profiles of unperturbed atomic resonances.

$< \gamma$ . In this case, the  $\Lambda$ -scheme approximation appears to be fairly reliable.

Figure 3 shows the absorptive and dispersive variations of the medium susceptibility for the detuning of the control mode  $\Delta = -50\gamma$  and in the region of detuning the probe mode also in the region of  $\bar{\Delta} \sim -50\gamma$ . For comparison, we present the result of calculation in the approximation of the  $\Lambda$ -scheme retaining only one transition  $|m'\rangle \rightarrow |n\rangle$ . The observed significant difference between the presented dependences for the exact and approximate calculations is related to a fundamental role played by the hyperfine interaction for probing the medium by two orthogonal polarization modes. If the hyperfine interaction were absent or negligibly small, then the system under consideration would correspond to the atomic transition between the ground state with the angular momentum  $j_0 = 1/2$  and

excited state also with the angular momentum  $j = 1/2$  [17]. Observation of the effects of alignment ( $\Lambda$ -coupling of the lower Zeeman states with  $\Delta M = \pm 2$ ) would be impossible. As can be seen from the presented dependences, in the vicinity of the control mode, the resonant Autler–Townes peak related to this coupling decreases with increasing detuning in the wings of the  $D_1$  transition. Under the same conditions, the calculation performed in terms of the isolated  $\Lambda$ -scheme yields a constant value of the resonance amplitude regardless of the detuning  $\Delta$ . The only quantity that changes is the width of the resonance.

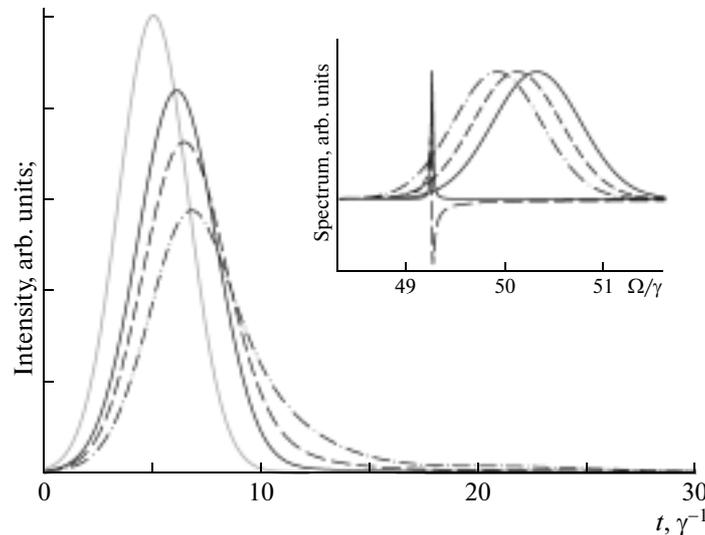
The situation drastically changes when the frequency of the control mode is tuned between the hyperfine components of the excited state, as is shown in Fig. 4 for  $\Delta = +50\gamma$ . In this case, the effect exhibits a certain enhancement as compared with predictions of the  $\Lambda$ -approximation. This phenomenon is particularly noticeable for the dispersive component of the susceptibility, which is more sensitive to the effects of



**Fig. 4.** Same as in Fig. 2, but for the detuning of control mode  $\Delta = +50\gamma$  and Rabi frequency  $\Omega_c = 15\gamma$ . Structure of Autler–Townes resonance is only shown in vicinity of  $\bar{\Delta} \sim \Delta$ . The structure of other components of the triplet is close to profiles of unperturbed atomic resonances.

interference related to the hyperfine interaction (see Eqs. (5) and (7)). The interrogation of the optically dense atomic medium in the vicinity of this resonance may lead to significantly more pronounced manifestations of the probe pulse delay. This may improve the efficiency of the quantum memory based on the Raman scattering protocol [3, 13, 15].

Figure 5 shows the results of calculations that demonstrate the delay of a pulse whose carrier frequency is varied in the vicinity of the Autler–Townes resonance peak corresponding to the detuning of the control field  $\Delta = +50\gamma$ . The calculations were performed for a homogeneous medium with the atomic concentration  $n_0$  and length  $L$ . In the case under consideration, the optical thickness of the medium is characterized by the cooperative parameter  $n_0(\lambda/2\pi)^2L$ , which is convenient for studying cooperative effects in one-dimensional systems. For the presented dependences, we assumed that  $n_0(\lambda/2\pi)^2L = 25$ , which is achievable for the experimental conditions in the case of atomic ensemble in a magneto-optical trap at a temperature below the Doppler limit of cooling. As the results of the above calculations show, the pulse delay effect appears to be quite noticeable. For the system with a transmissivity of 90%, it is possible to store in the memory channels and to retrieve about 40% of the pulse. It is also noteworthy that the efficiency of retrieval can be substantially improved if, at the stage of pulse restoration, the direction of propagation of the control pulse is inverted with respect to that used at the stage of recording. This possibility was discussed in [15].



**Fig. 5.** Dynamics of the probe pulse with a Gaussian profile transmitted through an optically dense atomic medium with the cooperative parameter  $n_0(\lambda/2\pi)^2L = 25$ . The dependences correspond to the control mode detuning  $\Delta = +50\gamma$  and to the Rabi frequency  $\Omega_c = 15\gamma = 15$ . The input profile is shown by a thickened curve; the output pulse profile varies with detuning of its carrier frequency. The inset shows position of spectral profile of the pulse with respect to the Autler–Townes resonance for the appropriate detunings. For the greatest detuning, the transmissivity is close to 90%, and about 40% of the pulse is accessible for restoration with the memory protocol.

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