Coherent light scattering from a quantum degenerate Bose-gas

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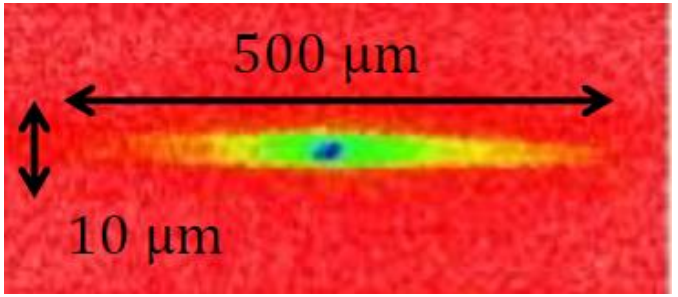
РОССИЙСКИЙ ФОНД ФУНДАМЕНТАЛЬНЫХ ИССЛЕДОВАНИЙ
РФФИ

Outline

- Motivation: Description of light scattering from complex quantum systems of matter
- Quantum scattering theory applied for light scattering from a degenerate Bose-gas (BEC)
- Infinite medium: polariton dynamics
- Calculation results: spectral behavior of the transmission and reflection coefficients in 1D-geometry
- Conclusion

Towards light-matter interface in complex quantum systems

R.G. Olave et al arXiv:1406.5783 (2014)



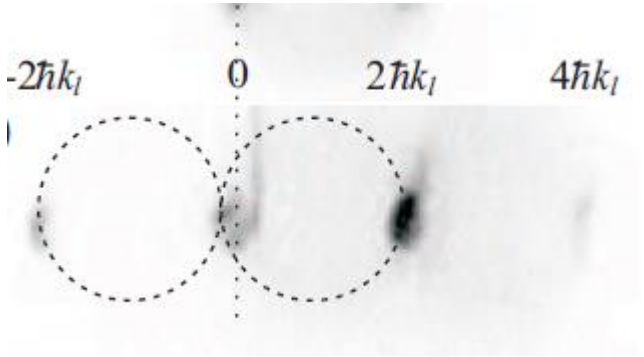
Atoms in a dipole trap: Raman based QM at high density

~100 μK

Lowering temperature

~100 nK

A. Hilliard et al Phys. Rev. A **78**, 051403(R)(2008)



BEC: Bragg grading and self organized structure correlated with backscattered light; coherent joint propagation of matter & light

Earlier predictions - H.D. Politzer Phys. Rev. A 43, 6414(R) (1991)

Superradiant Bose-Einstein Condensate

D. Schneble et al, Science 300, 475 (2003)

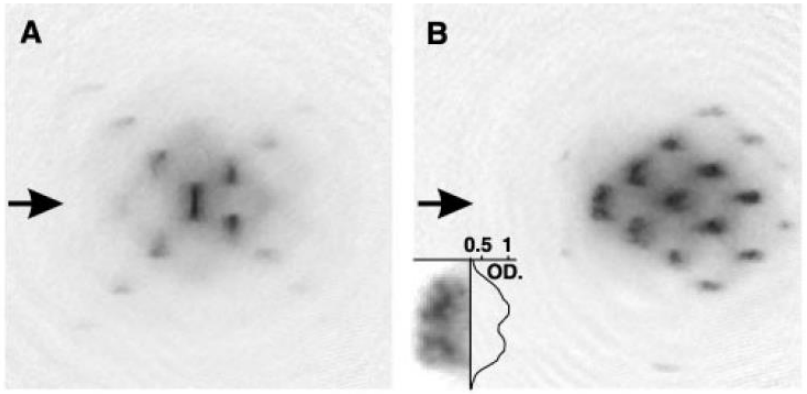


Fig. 1. Superradiant scattering of a laser beam (arrow) from a Bose-Einstein condensate in the short-pulse (**A**) and long-pulse (**B**) limit. Absorption images of the atomic density distribution were taken after 30 ms of ballistic expansion. In case A, Δ was -420 MHz and τ was 6 μs. In case B, Δ was -4400 MHz and τ was 800 μs. The inset in (B) is a magnification of the region around the condensate showing depletion of its center; for better visibility the contrast has been enhanced by a factor of 1.5. Also shown is a profile of the average optical density in the center region. The field of view of both images is 2.0 mm by 2.0 mm, and that of the inset is 120 μm by 270 μm.

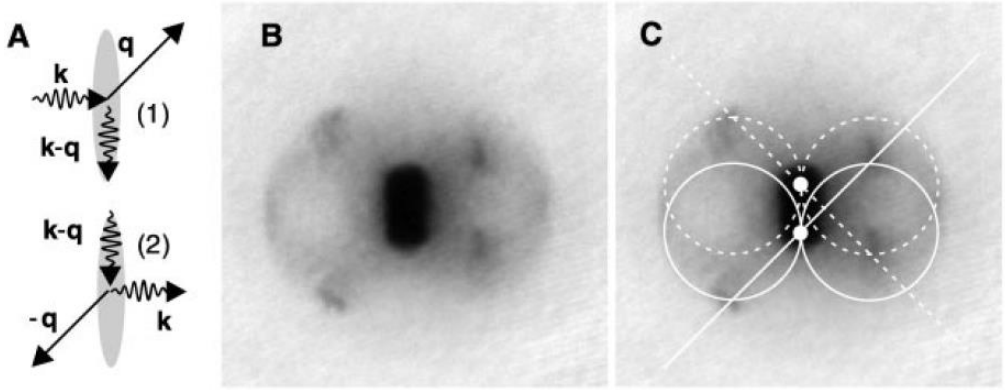
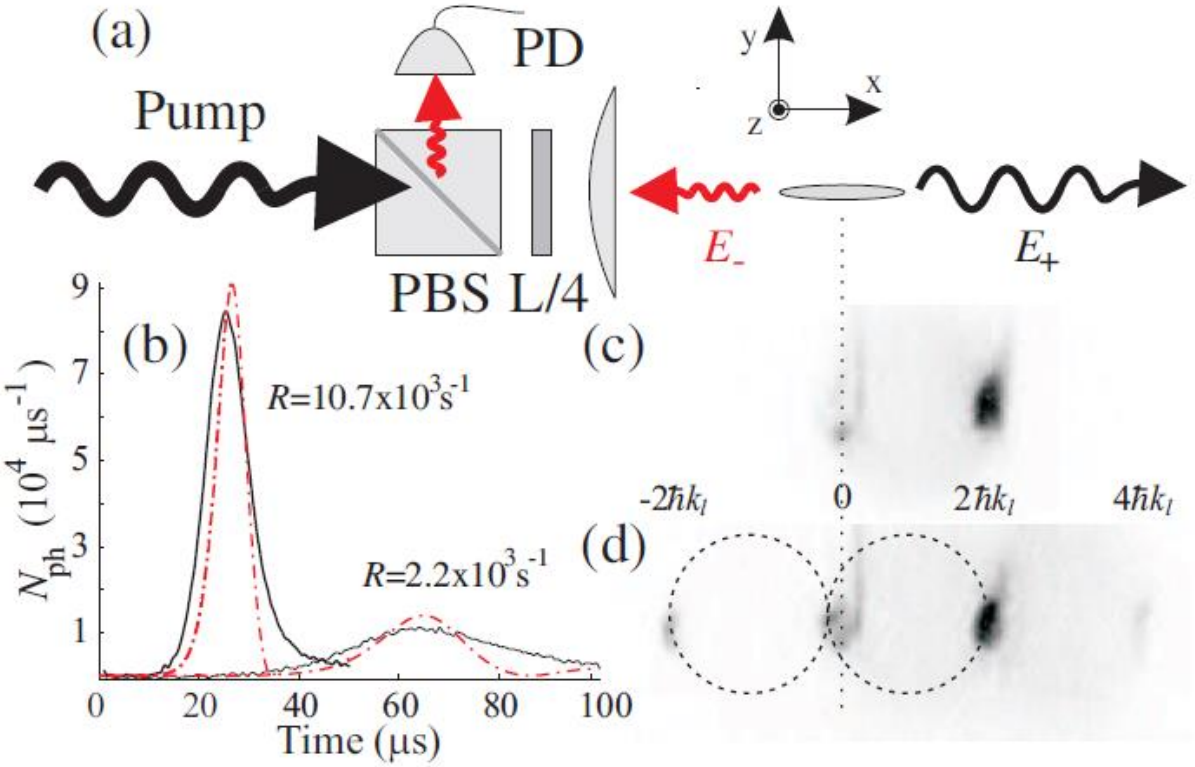


Fig. 2. Scattering processes (**A**) and scattering pattern (**B**) for a short ($\tau = 3$ μs) light pulse. The circles and lines in (**C**) illustrate the origin of the dipole emission halos and the Kapitza-Dirac diffraction orders.

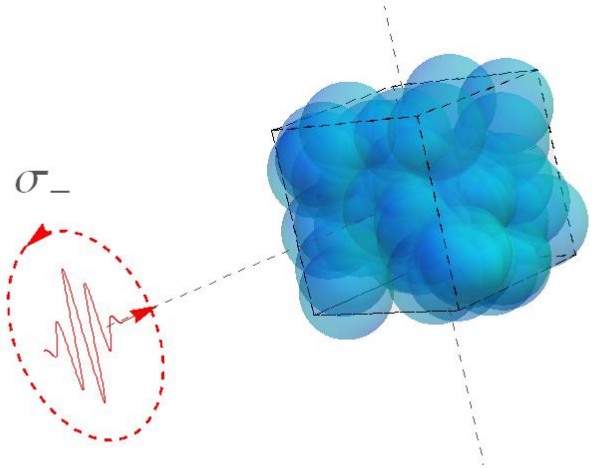
Self-organization via backscattering and Bragg-grading

A. Hilliard et al Phys. Rev. A 78, 051403(R) (2008)

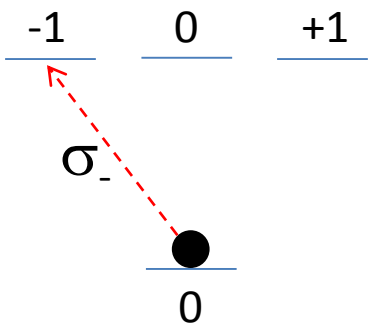
Detuning: $\Delta = -2\pi * 2.6$ GHz



BEC: Second quantized description



$$V = -\sum_n \int d^3r\, \mathbf{d}_{n0}\, \Psi_n^+(\mathbf{r})\Psi_0(\mathbf{r})\,\mathbf{E}(\mathbf{r})$$



$$\Psi_0(\mathbf{r})\left|\mathrm{BEC}\right\rangle^N = \Xi(\mathbf{r})\left|\mathrm{BEC}\right\rangle^{N-1} \qquad n_0 \sim \left|\Xi(\mathbf{r})\right|^2 \sim \text{const}$$

GP time dynamics:

$$i\hbar\frac{\partial}{\partial t}\Xi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\Delta\,\Xi(\mathbf{r},t) + U_0\left[\Xi(\mathbf{r},t)\left|\Xi(\mathbf{r},t)\right|^2 - n_0\Xi(\mathbf{r},t)\right]$$

$$\Xi(\mathbf{r},t) = \sqrt{n(\mathbf{r},t)}\exp\left[i\Phi(\mathbf{r},t)\right]$$

Local velocity:

$$\mathbf{v}_c = \frac{\hbar}{m}\nabla\Phi$$

Quantum scattering problem

T-matrix : $$T(E) = V + V\frac{1}{E-H}V$$

Differential cross section: $$\frac{d\sigma_{i\to f}}{d\Omega} = \frac{\mathbb{V}^2\omega'^2}{4\pi^2\hbar^2c^4}\left|T_{\underbrace{g'e'\mathbf{k}'}_{f},\underbrace{ge\mathbf{k}}_{i}}(E_i + i\cdot 0)\right|^2$$

Optical theorem: $$\sigma_i^{\text{tot}} = \sum_f \int \frac{d\sigma_{i\to f}}{d\Omega} d\Omega = -\frac{2\mathbb{V}}{\hbar c}\,\text{Im}[T_{i\to i}]$$

Interaction (dipole gauge):

$$V = -\sum_{a=1}^{N} \mathbf{d}^{(a)}\mathbf{E}(\mathbf{r}_a) \Rightarrow -\sum_n \int d^3r\, \mathbf{d}_{n0}\Psi_n^+(\mathbf{r})\Psi_0(\mathbf{r})\mathbf{E}(\mathbf{r})$$

Microscopic displacement field:

$$\mathbf{E}(\mathbf{r}) = \sum_{\mathbf{k}\alpha}\left(\frac{2\pi\hbar\omega_k}{\mathbb{V}}\right)^{1/2}\left[\mathrm{i}\mathbf{e}_{\mathbf{k}\alpha}a_{\mathbf{k}\alpha}e^{i\mathbf{kr}} - \mathrm{i}\mathbf{e}^*_{\mathbf{k}\alpha}a^+_{\mathbf{k}\alpha}e^{-i\mathbf{kr}}\right] = \mathbf{E}_\perp(\mathbf{r}) + 4\pi\mathbf{P}_\perp(\mathbf{r})$$

T-matrix vs. single-particle Green's function

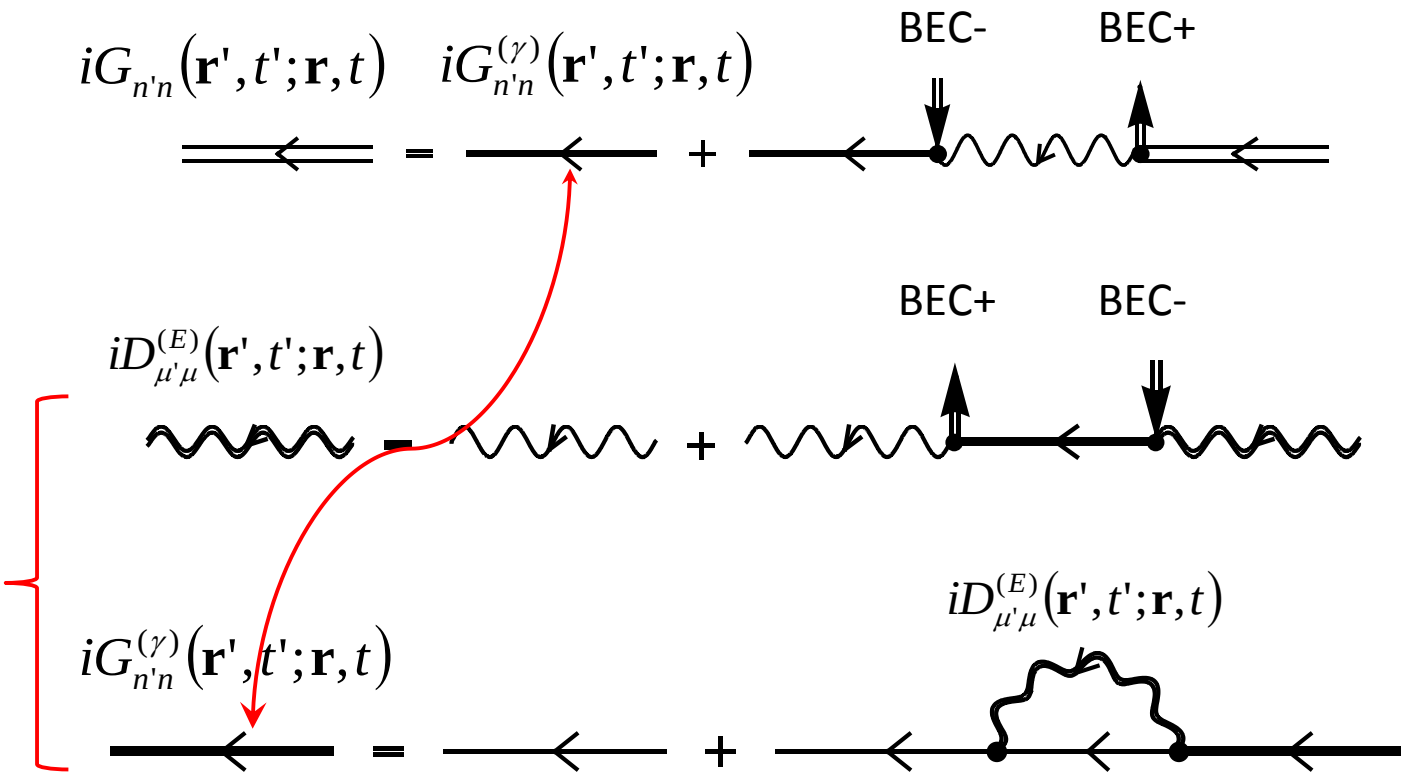
$$T(E) = V + V\frac{1}{E-H}V \qquad V = -\sum_n \int d^3r\, \mathbf{d}_{n0}\Psi_n^+(\mathbf{r})\Psi_0(\mathbf{r})\,\mathbf{E}(\mathbf{r})$$

Elastic channel with losses (Optical theorem):

$$T_{g\mathbf{e}'\mathbf{k}',g\mathbf{e}\mathbf{k}}(E) = \frac{2\pi\omega_k}{\mathbb{V}} \iint d^3r'\,d^3r \sum_{n',n} (\mathbf{d}\mathbf{e}')^*_{n'0}(\mathbf{d}\mathbf{e})_{n0}\, \mathrm{e}^{-i\mathbf{k}'\mathbf{r}'+i\mathbf{k}\mathbf{r}}$$
$$\times(-i)\int_0^\infty dt\, \mathrm{e}^{\frac{i}{\hbar}(E-E_0)t-0\cdot t}\, \Xi^*(\mathbf{r}')\Xi(\mathbf{r})\, iG_{n'n}(\mathbf{r}',t;\mathbf{r},0)$$

$$iG_{n'n}(\mathbf{r}',t';\mathbf{r},t) = \left\langle \mathrm{BEC}^{N-1} \middle| T\Psi_{n'}(\mathbf{r}',t')\,\Psi_n^+(\mathbf{r},t) \middle| \mathrm{BEC}^{N-1} \right\rangle$$

Diagram expansion of the Green's function



The SE-part associated with incoh. scattering

$$\begin{cases} D^{(E)}_{\mu\mu'}(\mathbf{k},\omega) = -\dfrac{4\pi\hbar\dfrac{\omega^2}{c^2}}{k^2 - \underbrace{\left[1-\dfrac{4\pi}{\hbar}d_0^2 n_0 G^{(\gamma)}_{nn'}(\hbar k,\hbar\omega)\right]}_{\cong\,\varepsilon(\omega)}\dfrac{\omega^2}{c^2}}\left\{\delta_{\mu\mu'} - \dfrac{k_\mu k_{\mu'}}{\underbrace{\left[1-\dfrac{4\pi}{\hbar}d_0^2 n_0 G^{(\gamma)}_{nn'}(\hbar k,\hbar\omega)\right]}_{\cong\,\varepsilon(\omega)}\dfrac{\omega^2}{c^2}}\right\} \\ \left[E - \dfrac{p^2}{2m} - E_n - \Sigma(p,E)\right]G^{(\gamma)}(p,E) = \hbar \end{cases}$$

I.E. Dzyaloshinskii, L.P. Pitaevskii 1959

$$\Sigma(p,E) = -\frac{4\pi}{3}d_0^2\frac{\omega^2}{c^2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2 - \underbrace{\left[1-\dfrac{4\pi}{\hbar}d_0^2 n_0 G^{(\gamma)}_{nn'}(\hbar k,\hbar\omega)\right]}_{\cong\,\varepsilon(\omega)}\dfrac{\omega^2}{c^2}}\left\{3 - \frac{k^2}{\underbrace{\left[1-\dfrac{4\pi}{\hbar}d_0^2 n_0 G^{(\gamma)}_{nn'}(\hbar k,\hbar\omega)\right]}_{\cong\,\varepsilon(\omega)}\dfrac{\omega^2}{c^2}}\right\}$$

$$\Sigma(p,E) \cong -\frac{8\pi}{3}d_0^2\frac{\omega^2}{c^2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2 - \varepsilon(\omega)\dfrac{\omega^2}{c^2}} + \frac{4\pi}{3}d_0^2\int\frac{d^3k}{(2\pi)^3}\frac{1}{\varepsilon(\omega)}$$

Renormalization of the self-energy

(i) Static field contribution:

$$\frac{4\pi}{3} d_0^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{\varepsilon(\omega)} = -\mathbf{d}\,\mathbf{E}_{\mathrm{dip}}^{(\mathrm{m})}(\mathbf{0})$$

<= subtract

Dipole self-energy

$$E_{\mathrm{self}} = \frac{4\pi}{3} d_0^2 \int \frac{d^3k}{(2\pi)^3} = -\mathbf{d}\,\mathbf{E}_{\mathrm{dip}}^{(\mathrm{vac})}(\mathbf{0})$$

$$-\mathbf{d}\,\mathbf{E}_{\mathrm{dip}}^{(\mathrm{med})}(\mathbf{0}) + \mathbf{d}\,\mathbf{E}_{\mathrm{dip}}^{(\mathrm{vac})}(\mathbf{0})$$

Int. with environment=>

$$-\mathbf{d}\left(\mathbf{E}_{\mathrm{loc}}(\mathbf{0}) = \frac{4\pi}{3} n_0 \mathbf{d}\right) = -\frac{4\pi}{3} n_0 d_0^2$$

(ii) Radiation part:

$$-\frac{8\pi}{3} d_0^2 \frac{\omega^2}{c^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 - \varepsilon(\omega)\frac{\omega^2}{c^2}}$$

<= add

= 0

$$\frac{8\pi}{3} d_0^2 \frac{\omega^2}{c^2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2 - \frac{\omega^2}{c^2} - \mathrm{i}0} + \left(\hbar\Delta_{\mathrm{Lamb}}^{(\mathrm{vac})} => \infty\right) - \mathrm{i}\hbar\frac{\gamma}{2}$$

$$\left(\hbar\Delta_{\mathrm{Lamb}}^{(\mathrm{vac})} => \infty\right) - \mathrm{i}\hbar\sqrt{\varepsilon(\omega)}\frac{\gamma}{2}$$

<= Radiation damping + SC-corrected Lamb shift

I. M. Sokolov et al., Phys. Rev. A 79 053405 (2009)

The closed self-consistent equation for the complete single particle Green's function

$$\left[E+\frac{\hbar^2}{2m}\Delta_{\mathbf{r}}-\hbar\omega_0+\frac{4\pi}{3}n_0\,d_0^2+\mathrm{i}\hbar\sqrt{\varepsilon(\omega)}\,\gamma/2\right]G_{nn'}(\mathbf{r},\mathbf{r}';E)$$

$$-\frac{1}{\hbar}\sum_{n''}\sum_{\mu,\mu'}\int d^3r''\left(d^{\mu}\right)_{n0}\left(d^{\mu'}\right)_{0n''}D^{(E)}_{\mu\mu'}(\mathbf{r},\mathbf{r}'',\omega=E/\hbar)\,\Xi(\mathbf{r})\Xi^{*}(\mathbf{r}'')\,G_{n''n'}(\mathbf{r}'',\mathbf{r}';E)=\hbar\delta_{nn'}\,\delta(\mathbf{r}-\mathbf{r}')$$

Electric field causal-type propagator in free space (Dipole gauge):

$$D^{(E)}_{\mu\mu'}\left(\mathbf{r},\mathbf{r}'',\omega=\frac{E}{\hbar}\right)=-\delta_{\mu\mu'}\frac{\hbar\omega^2}{c^2}\frac{1}{R}\,\mathrm{e}^{i\frac{|\omega|}{c}R}-\frac{\partial^2}{\partial X_{\mu}\partial X_{\mu'}}\left[\frac{\hbar}{R}\mathrm{e}^{i\frac{|\omega|}{c}R}\right]\qquad R=\left|\mathbf{r}-\mathbf{r}''\right|$$

The equation implies that density component n_0 as well as the dielectric constant $\varepsilon(\omega)$ can be spatially dependent quantities

Infinite medium: Polariton dynamics

$$G_{nn'}(\mathbf{k},\omega) = G_{\parallel}(k,\omega)\frac{k_n k_{n'}}{k^2} + G_{\perp}(k,\omega)\left[\delta_{nn'} - \frac{k_n k_{n'}}{k^2}\right] \qquad \Xi(\mathbf{r})\Xi^*(\mathbf{r}'') \approx n_0$$

Transverse component: polariton-type excitation wave

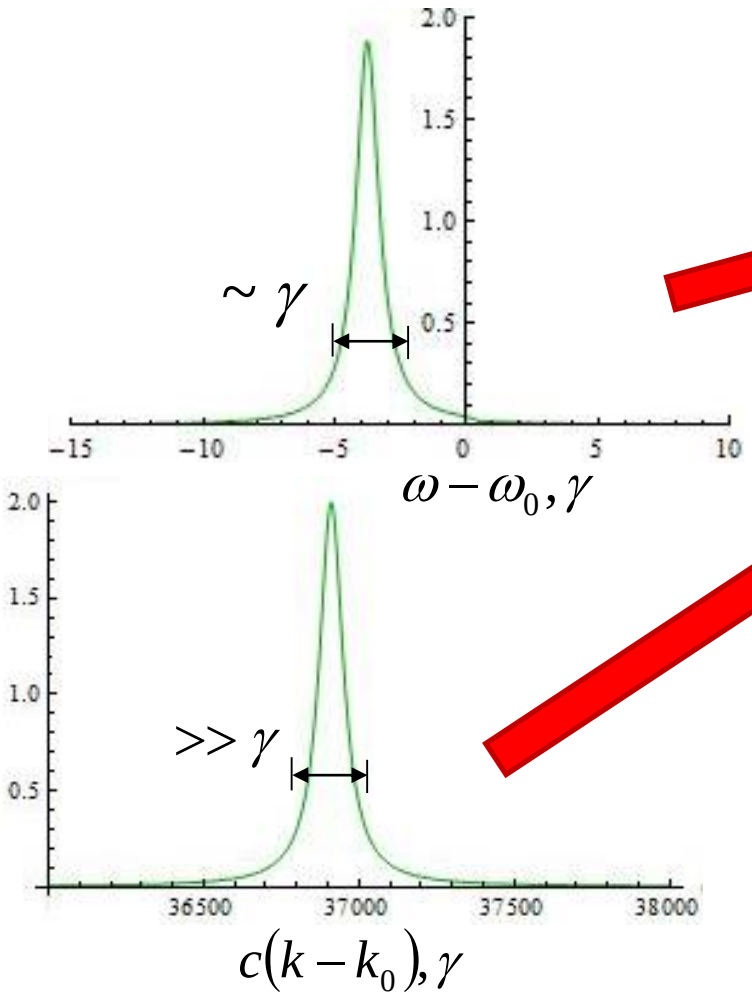
$$G_{\perp}(\mathbf{k},\omega) = \left\{ \underbrace{\omega - \omega_0 - \frac{\hbar k^2}{2m} + \frac{4\pi n_0 d_0^2}{3\hbar} + \frac{i\sqrt{\varepsilon(\omega)}\gamma}{2}}_{\text{Atom/exciton-type}} - \underbrace{\frac{4\pi n_0 d_0^2 \omega^2}{\hbar\left(\omega^2 - c^2 k^2\right)}}_{\text{Photon-type}} \right\}^{-1}$$

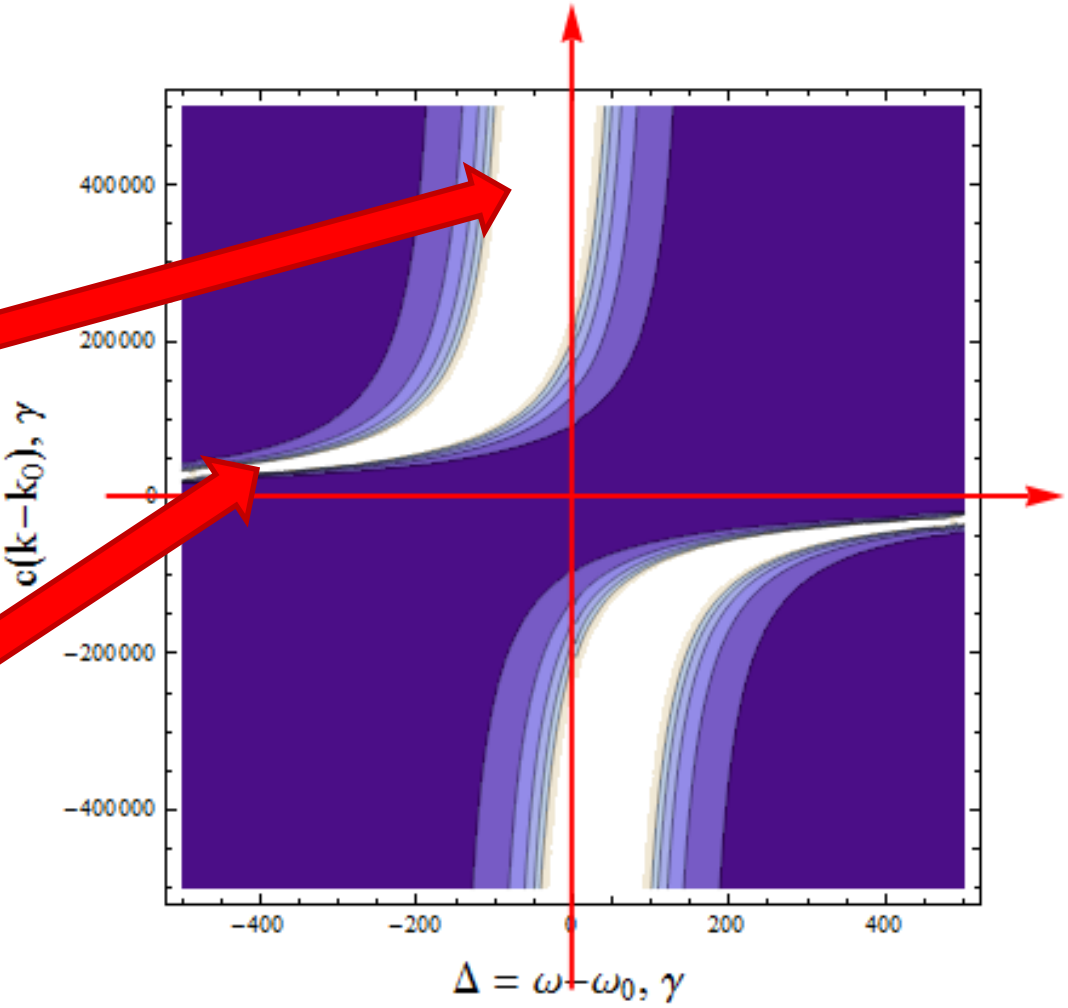
Longitudinal component: exciton-type dynamics

$$G_{\parallel}(\mathbf{k},\omega) = \left\{ \omega - \omega_0 - \frac{\hbar k^2}{2m} - \frac{8\pi n_0 d_0^2}{3\hbar} + \frac{i\sqrt{\varepsilon(\omega)}\gamma}{2} \right\}^{-1}$$

Polariton dynamics

$$-\mathrm{Im}\,G_{\perp}(\mathbf{k},\omega)$$

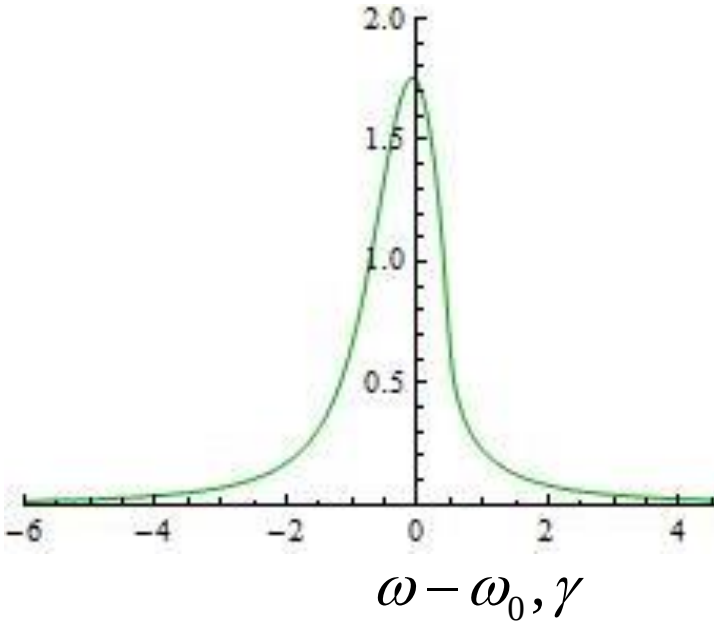


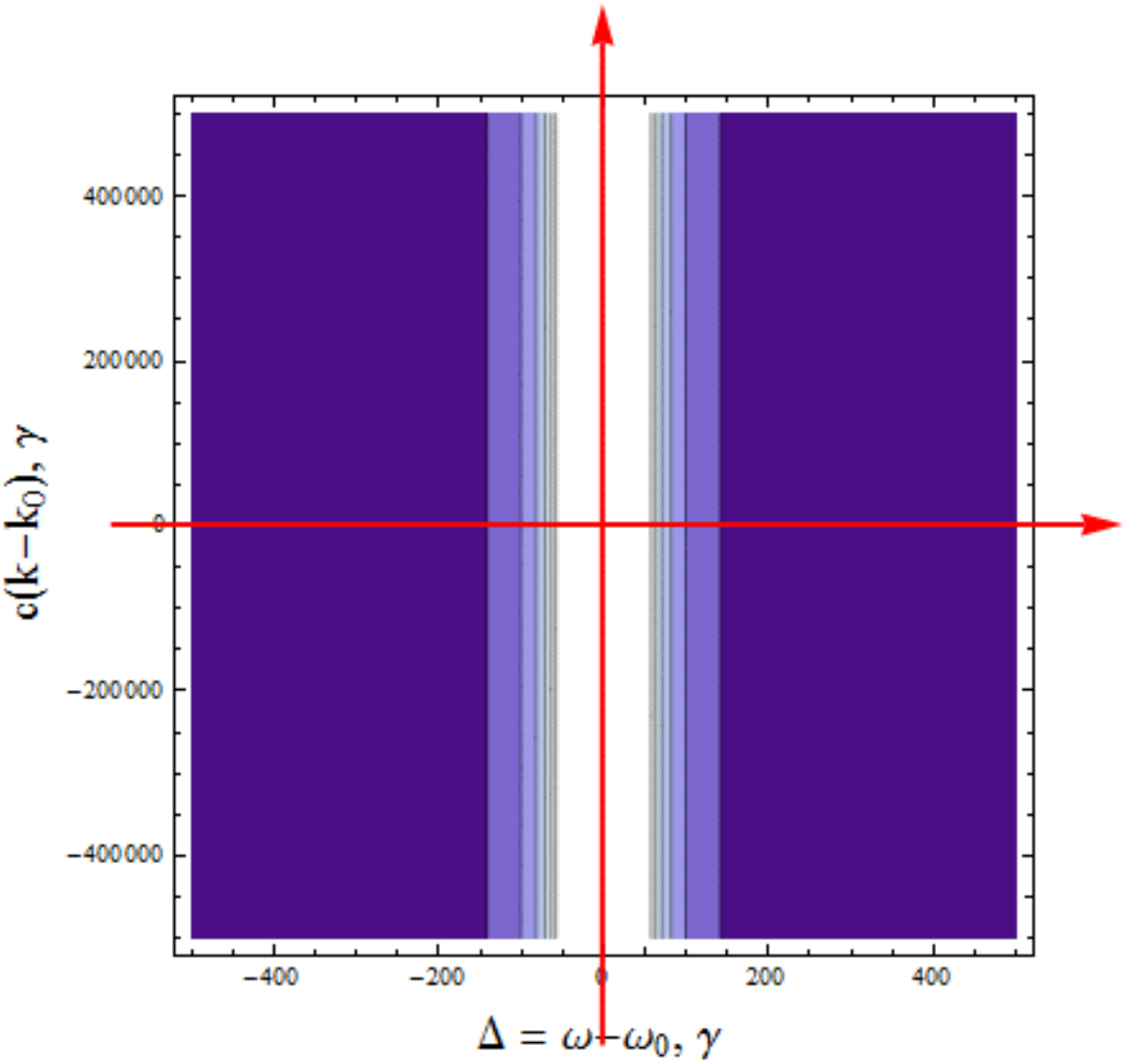


V. Ezhova et al. arXiv:1602.07562

Longitudinal component (exciton-type)

$$-\operatorname{Im} G_{\parallel}(\mathbf{k},\omega)$$





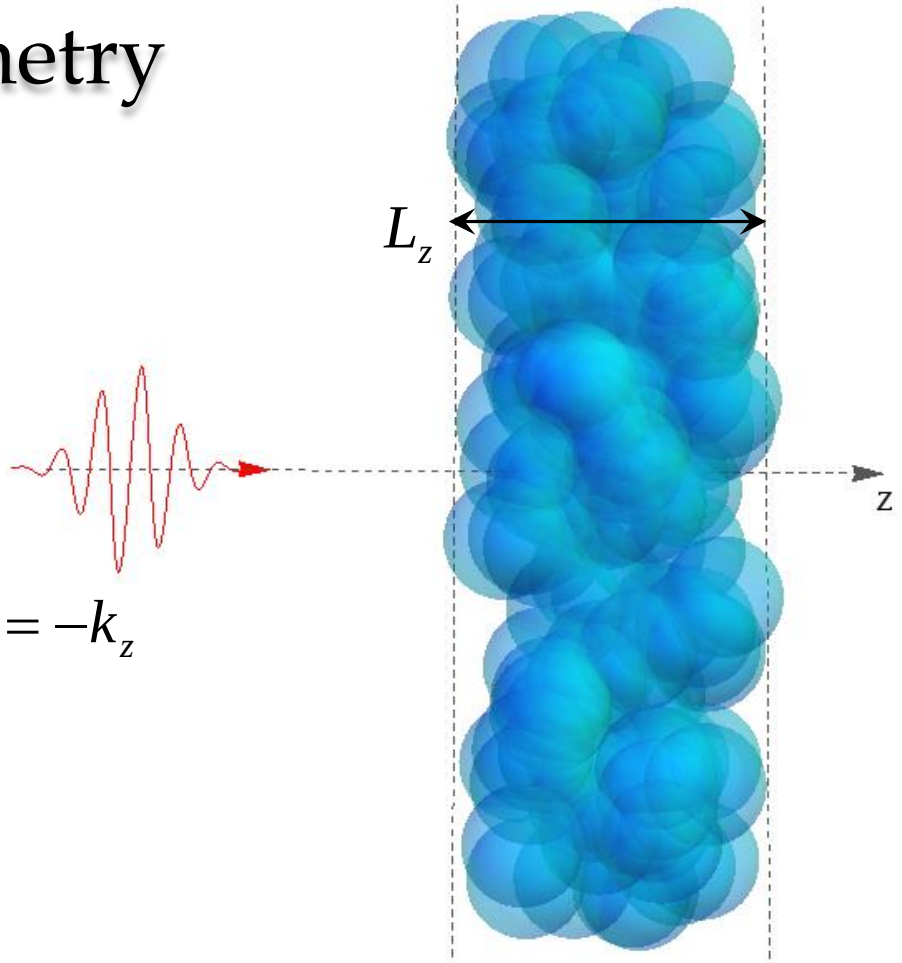
1D-Scattering: slab geometry

Transmission coefficient

$$\mathrm{T} = \left|S_{ii}\right|^2 = \left|1 - i\frac{\mathbb{L}}{\hbar c} T_{\underbrace{gek_z}_{i},\underbrace{gek_z}_{i}}(E_i + i\cdot 0)\right|^2$$

Reflection coefficient

$$\mathrm{R} = \left|S_{i'i}\right|^2 = \left|i\frac{\mathbb{L}}{\hbar c} T_{\underbrace{gek'_z}_{i'},\underbrace{gek_z}_{i}}(E_i + i\cdot 0)\right|^2 \quad k'_z = -k_z$$

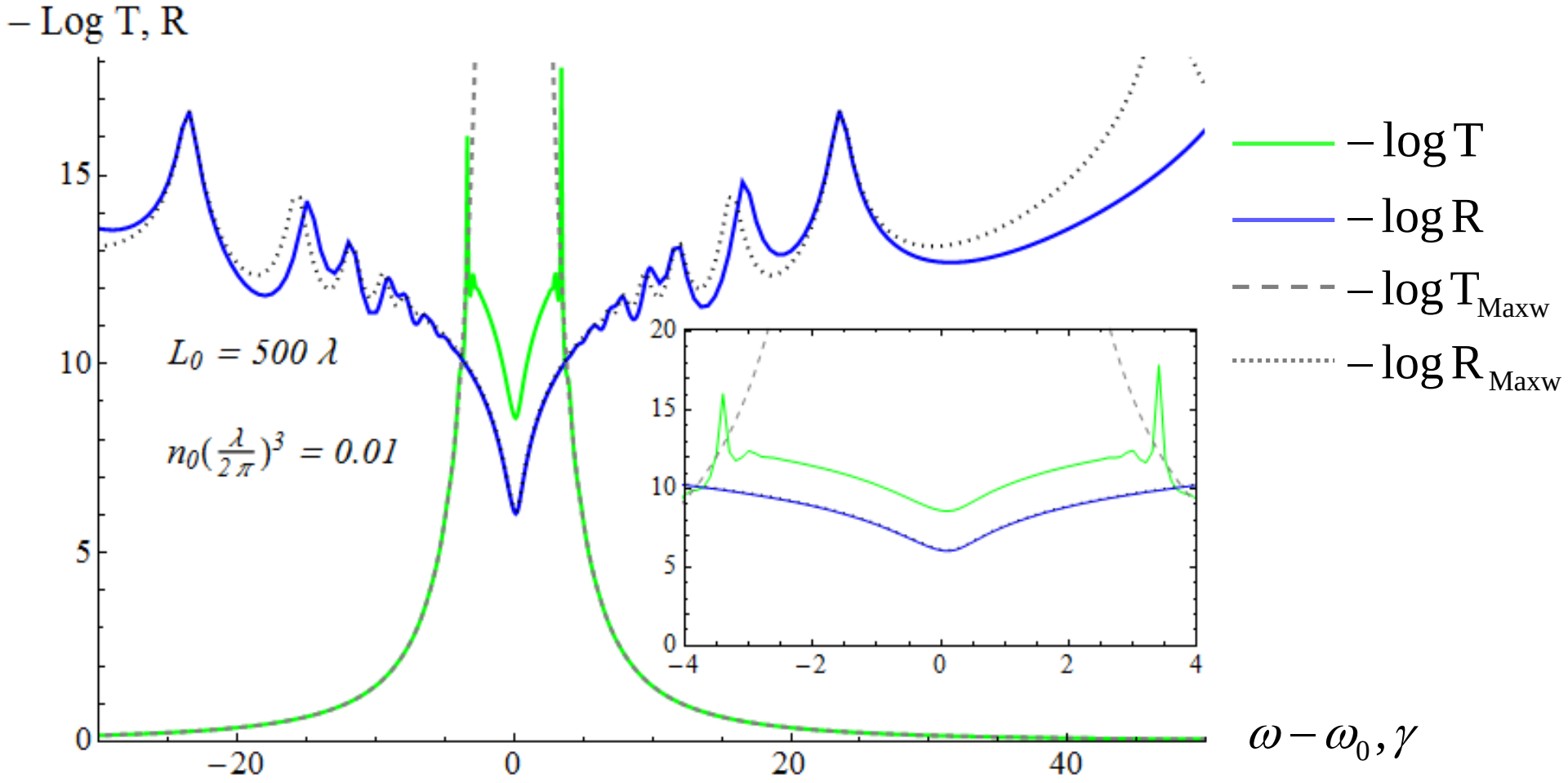


$k_z = 2\pi m / L_z$, where **m=0,+-1,+-2,….**

$$\left[E + \frac{\hbar^2 k_z^2}{2m} - \hbar\omega_0 + \frac{4\pi}{3} n_0 d_0^2 + \mathrm{i}\hbar\sqrt{\varepsilon(\omega)}\gamma/2\right] G_{nn'}(k_z, k'_z; E)$$

$$-\frac{1}{\hbar}\sum_{n''}\sum_{\mu,\mu'}\sum_{k''_z} n_0 \left(d^{\mu}\right)_{n0}\left(d^{\mu'}\right)_{0n''} D^{(E)}_{\mu\mu'}(k_z, k''_z, \omega) G_{n''n'}(k''_z, k'_z; E) = \hbar L_z \delta_{nn'}\delta_{k_z k'_z}$$

Transmission/Reflection spectral dependence



Conclusion

- We have applied subsequent formalism of the quantum scattering theory to the problem of light scattering from a degenerate quantum Bose-gas (BEC)
- Light propagates through BEC as a superposed state of field and matter as a polariton-type excitation wave
- In 1D-geometry we have performed spectra of the transmission and reflection coefficients calculated in a vicinity of the atomic resonance
- Further work: complex quantum systems, such as degenerate quantum gas, can be promising objects for developing quantum interface between light and matter

Thank you for your attention!